What's My Impedance?

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I. Introduction

Microwave circuit simulators have models for all the traditional transmission line cross-sections such as microstrip, stripline and coplanar-waveguide (CPW). However, there are cases that the standard models do not cover. This is particularly true in multilayer environments such as MMIC's, packages and printed circuit boards (PCB's).

In all of these environments, we have the freedom to design transmission lines that may not fit any of the conventional definitions. We may need microstrip or CPW with a dielectric overlay for an MMIC or package. In a multilayer PCB, we may have a buried stripline conductor with grounded isolations strips in close proximity and upper/lower ground planes that are not symmetrically located. This configuration is some kind of stripline/CPW hybrid depending on the strip widths and the various ground plane spacings. For any strip configuration, the effects of strip thickness and the actual strip cross-section due to etching or other processing are also of interest.

Fortunately, some inexpensive electromagnetic field-solvers can help us compute the impedance and effective dielectric constant of arbitrary transmission line cross-sections. For good engineering results, our goal is 1% to 2% accuracy. These tools also give us the freedom to ask many interesting "what if" questions regarding the configuration we have chosen.

II. Types of Field-Solvers

The use of electromagnetic field-solvers in microwave circuit design has increased dramatically in the last decade. We can divide these tools into three broad classes. We characterize each class not by the numerical method used but rather by the order of the geometry they can analyze. Within each class, any number of different numerical methods may be used. Model building time, numerical effort and solution time all increase dramatically as the geometry gets more complex.

The type of field-solver we will focus on in this work is the 2D cross-section solver, Fig. 1(a). This solver is suitable for strips or slots with uniform cross-section going into the page. For problems with one or two signal conductors, it is quite easy to compute the impedances and phase velocities of each mode. Numerically, we only have to consider a small, bounded 2D region, so solution time generally will not be an issue. Many of these field-solvers are stand-alone tools while some are integrated within a linear/non-linear simulator. Table I is a partial listing of 2D field-solvers.
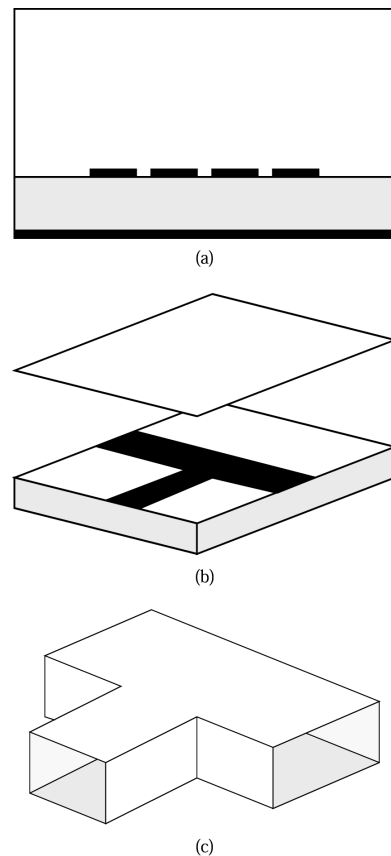

Fig. 1. Field-solvers classified by order of geometry; (a) 2D cross-section, (b) 2.5D planar, (c) 3D arbitrary.

If we want to solve more general planar circuits, we generally move to a 2.5D planar-solver, Fig. 1(b). These tools are also called 3D mostly planar solvers by some software vendors. With these tools, an arbitrary number of homogeneous dielectric layers are allowed. An arbitrary planar metal pattern can then be placed at the interface between any pair of dielectric layers. Via metal can also be used to connect metal layers. This is where the half

dimension comes from in the 2.5D description; we are somewhere in between a strictly planar structure and an arbitrary 3D structure. There are two fundamental numerical formulations for this type of problem, one that assumes the circuit is in a closed, metallic box and one where no box walls are present. Numerically, the software is looking for the unique current distribution on the strips that forces the tangential component of the E-field on the strips to be zero. Compared to the 2D cross-section-solvers, the numerical effort has increased dramatically and solution time becomes an issue.

TABLE I
PARTIAL LIST OF 2D FIELD-SOLVERS

Stand-Alone Software – PDE Solvers
FlexPDE – PDE Solutions
QuickField – Tera Analysis
FEMLAB – COMSOL
Stand-Alone Software – Dedicated Static Solvers
Maxwell SI 2D – Ansoft
LINPAR and MULTLIN – Artech House
ApsimRLGC – Applied Simulation Technology
ELECTRO – Integrated Engineering Software
ElecNet – Infolytica
Opera-2D – Vector Fields
Integrated with linear/non-linear simulator
MCPL model – Ansoft *Serenade*
VUSTLS model – AC Microwave *LINMIC+/N*
MLnCTL model – Agilent *ADS*

Finally, we come to the 3D field-solvers, which allow us to analyze a truly arbitrary 3D structure, Fig. 1(c). The basic formulation for these solvers assumes a closed, metallic boundary around the solution region. However, an open environment can be approximated using various types of absorbing boundaries. While these tools offer great flexibility, the penalty is modeling time and solution time. Building a model in 3D is considerably more difficult than 2D or 2.5D modeling. Numerically, we are forced to solve for the fields in the entire 3D volume, which leads to a dramatic increase in solution time compared to the other two classes of solvers.

For any given problem, one of the three general solver types will offer the most efficient solution. In the course of a design project, we might use all three types of solver at some point. However, in the very early stages of a project, we are typically choosing substrate materials and developing some intuition for the range of impedances that we can realize. We might also be experimenting with non-standard transmission line cross-sections. A fast computation of impedance using an inexpensive 2D cross-section solver is quite valuable at this stage.

III. Computing Z_o and ε_{eff}

The two most basic parameters that define a transmission line are characteristic impedance, Z_o and the velocity of propagation, v_p. For single strips and coupled pairs it is easy to compute these parameters using a stand-alone 2D field-solver. Characteristic impedance and velocity of propagation are defined in terms of inductance per unit length, L and capacitance per unit length, C.

$$Z_o = \sqrt{\frac{L}{C}} \tag{1}$$

$$v_p = \frac{1}{\sqrt{LC}} \tag{2}$$

These equations are for lossless lines and ignore skin depth effects [1]. At first glance, these equations also imply that we need two different 2D cross-sections solvers, an electrostatic solver to compute C and a magnetostatic solver to compute L. We can simplify the problem if we take advantage of one special case where we know the phase velocity in advance. This special case is any air dielectric transmission line where the velocity must be the speed of light,

$$c = 2.998 \times 10^8 \text{ m/s.} \tag{3}$$

Substituting into the equation for velocity of propagation, we get

$$c = \frac{1}{\sqrt{LC_o}} \tag{4}$$

where C_o is the capacitance per unit length of the line when all of the dielectrics are air ($\varepsilon_r = 1$). If all the materials are also non-magnetic ($\mu_r = 1$) we can solve for L

$$L = \frac{1}{c^2 C_o} \tag{5}$$

and substitute back into the equations for Z_o and v_p

$$Z_o = \frac{1}{c\sqrt{CC_o}} \tag{6}$$

$$v_p = c\sqrt{\frac{C_o}{C}} \tag{7}$$

For mixed dielectric problems like microstrip, we can define an effective dielectric constant, ε_{eff} that is related to the actual velocity of propagation in the medium by

$$\varepsilon_{eff} = \left(\frac{c}{v_p}\right)^2 \tag{8}$$

Substituting in the equation for v_p as a function of C and C_o, we get

$$\varepsilon_{eff} = \frac{C}{C_o} \tag{9}$$

If the transmission line medium is homogeneous,

$$\varepsilon_r = \varepsilon_{eff} = \frac{C}{C_o} \qquad (10)$$

If we solve for C_o and substitute back into (6),

$$Z_o = \frac{\sqrt{\varepsilon_r}}{cC} \qquad (11)$$

Now we can solve for impedance and effective dielectric constant for any inhomogeneous medium by using only an electrostatic solver to find the two values of capacitance per unit length. The first computation is for C, the capacitance per unit length with all dielectrics present. The second computation is for C_o, the capacitance per unit length with all dielectrics removed. With these two values in hand, we use equations (6) and (9) to find Z_o and ε_{eff}. For homogeneous dielectrics we can use (11) to find Z_o.

IV. Solving for *C* and *C_o*

We will look at two stand-alone electrostatic solvers suitable for finding C and C_o. These tools are actually general-purpose solvers for partial differential equations (PDE's). They can be used for electrostatic and magnetostatic problems, thermal analysis and stress/strain problems among others. In our case, we are interested in solving Laplace's equation in two dimensions

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \qquad (12)$$

where $\phi(x, y)$ is the potential in the region of interest. The classic paper by Green [2] outlines the solution for several geometries using the finite difference method. This formulation is so simple it can also be implemented as a spreadsheet program [3].

If we are solving for a strip in a box (Fig. 2) we typically set the strip potential to 1V, the boundary to 0V and solve for the potential at a number of points inside the box.
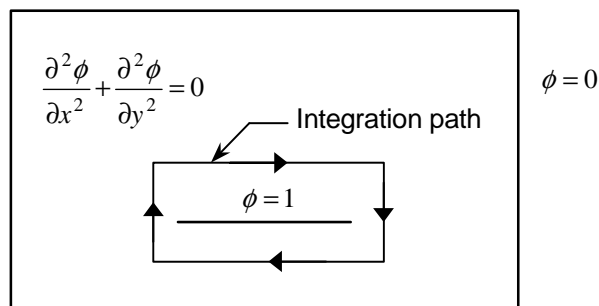

Fig. 2.   Generic strip in a box problem governed by Laplace's equation. The strip is set to 1V and the outer conductor to 0V.

If we then integrate along a closed path around the strip, we get the total charge on the strip in Coulombs per meter, which is what the software will typically report. If we remember that charge equals capacitance times voltage,

$$Q = CV \qquad (13)$$

then we can interpret the results from the software directly as capacitance per meter.

Note that we are performing a static ($f = 0$) analysis of our transmission line structure. For pure TEM structures like stripline and coaxial lines, a static analysis will be accurate right up to the first higher order mode frequency. Quasi-TEM structures, like microstrip, will have frequency dependent impedance and effective dielectric constant. However, if the substrate is thin in terms of wavelength, and if the strip conductor is very narrow in terms of wavelength, then a static analysis should be perfectly adequate.

V. A Quick Look at *QuickField*

QuickField is a popular 2D PDE solver that uses the finite element method (FEM) [4]. A screen capture showing the finite element mesh for a simple microstrip in a box is shown in Fig. 3. An FEM code subdivides the region of interest with triangles (the mesh) and solves for the potential at the vertices (the nodes). *QuickField* allows the user to draw the desired geometry using a graphical user interface. Although this program does not have automatic mesh refinement, the circles at several vertices indicate the desired mesh size at these points chosen by the user. The vertex centered at the bottom of box is a "dummy" vertex, which forces more mesh in the high field region under the strip.

Some of the post-processing capability in *QuickField* is shown in Fig. 4. With 1V on the strip, we see constant potential contours, a false color plot of the potential and the closed integration path for computing the total charge on the strip. There is also a wizard to aid the user in capacitance calculations.

When using any numerical solution of Maxwell's equations we have to constantly test the convergence of our solution. Is the mesh shown in Fig. 3 "good enough" for 1% to 2% accuracy in impedance? Unfortunately, the only way to tell for sure is to increase the density of the mesh and solve the problem again. The student or shareware version used here is limited to 200 nodes in the FEM mesh. With only 200 nodes, it will be difficult to find a fully converged solution to this problem. If we apply symmetry, our limited number of nodes can be distributed over a smaller region.

VI. Flexible *FlexPDE*

Another inexpensive 2D solver for partial differential equations is *FlexPDE*. This software also uses FEM and offers the additional feature of automatic mesh refinement. *FlexPDE* uses a text file input rather than a graphical user interface. At first, this may seem cumbersome, but the advantage is you can "program" your geometry using variables and then make changes very rapidly. Once you get a few projects running, the input files become templates for new projects. The student version of *FlexPDE* is limited to 400 nodes.

For demonstration purposes, we will use a simple stripline geometry that should be very close to 50 ohms. Because stripline has an exact analytical solution, it is very useful as a test case [5]. The test case geometry, dimensions and impedance computed by *LineCalc* are shown in Fig. 5.
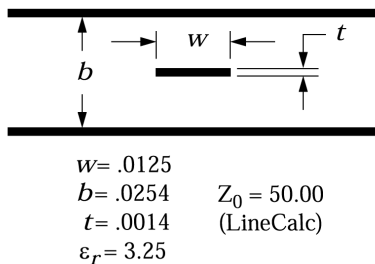

$w= .0125$
$b= .0254$ $Z_0 = 50.00$
$t= .0014$ (LineCalc)
$\varepsilon_r = 3.25$

Fig. 5. Geometry and dimensions (inch) for the simple stripline test case.

The result of the analysis run to find C is shown in Fig. 6. The output graphs include the FEM mesh, lines of constant potential, the normal component of the E-field along the integration path and normalized E-field vectors. The FEM mesh was automatically refined four times by the software. The numerical result for the contour integration is in the lower left corner of the last graph. In this case the dielectric is homogeneous, so $\varepsilon_{eff} = \varepsilon_r$ and we can use (11) to find Z_o.

$$C = 1.2117 \times 10^{-10} \text{ F/m} \qquad (14)$$

$$Z_o = \frac{\sqrt{\varepsilon_r}}{cC} = 49.63 \qquad (15)$$

At some point, we should test the convergence of our solution by plotting the computed impedance as a function of mesh refinement.

VII. Coupled Line Parameters Using Symmetry

Symmetry is a useful concept when we compute coupled line parameters. We could compute the complete two strip cross-section, or we can make use of the vertical line of symmetry in most coupled strip problems. Fig. 7 is a coupled stripline example with electrical parameters computed by *LineCalc*.

$w= .0117$
$s = .0064$ $Z_{even} = 59.99$
$b= .0254$ $Z_{odd} = 42.09$
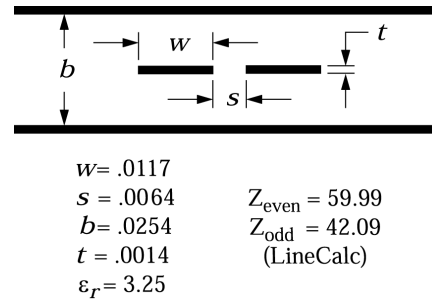$t = .0014$ (LineCalc)
$\varepsilon_r = 3.25$

Fig. 7. Geometry and dimensions (inch) for simple coupled stripline test case.

The even-mode has equal potentials on both strips with the same sign. We can place a vertical magnetic wall between the two strips without modifying the pattern of electric field lines, Fig. 8. The odd-mode has equal potentials with opposite signs on the two strips. A vertical electric wall between the two strips will not modify the pattern of electric field lines. The superposition of the even-mode and odd-mode excitations is equivalent to applying a 1V source to one of the strips with all other conductors at 0V.
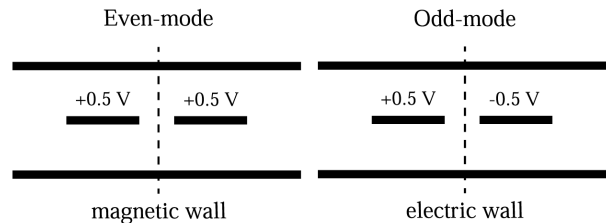

Fig. 8. Even-mode and odd-mode excitations for a coupled pair of strips.

If the dielectric were inhomogeneous, we would have to compute C and C_o for the even-mode and for the odd-mode, a total of four capacitance calculations. We will use equation (6) here, just for practice. For the even-mode analysis we put 0.5V on the strip, 0V on the outer conductor and a magnetic wall down the symmetry plane. Fig. 9 is a plot from *FlexPDE* of the E-field vectors for the even-mode, with all the lengths normalized to one value.
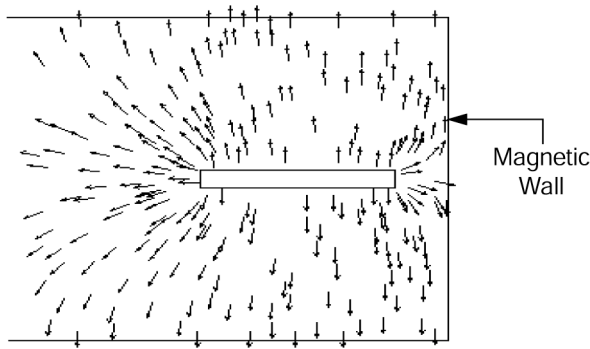
Fig. 9.   E-field vectors for the even-mode, with all the lengths normalized to one value.

The computed electrical parameters for the even-mode are shown below. Note the correction factor or "impedance multiplier" in the numerator of the equation for Z_{even}.

$$C = 4.9645 \times 10^{-11} \ \text{F/m} \tag{16}$$

$$C_o = 1.5275 \times 10^{-11} \ \text{F/m} \tag{17}$$

$$Z_{even} = \frac{0.5}{c\sqrt{CC_o}} = 60.56 \tag{18}$$

$$\varepsilon_{eff} = \frac{C}{C_o} = 3.25 \tag{19}$$

The computed even-mode impedance is within 1% of the analytical value computed by *LineCalc*. For microstrip, we would expect the even-mode ε_{eff} to be lower than ε_r. The calculations for the odd-mode proceed in a similar fashion.

For more than two strips above a ground plane, we can devise a procedure to build the C and C_o matrices using a general purpose PDE solver. However, at some point this becomes a very tedious process. For more than two conductors, the dedicated electrostatic solvers listed in Table I are probably more efficient for obtaining L and C in matrix form. If you need to optimize multiple microstrip or stripline conductors, the integrated solvers listed in Table I are probably the best choice. In addition, MCPL in *Serenade* and VUSTLS in *LINMIC+/N* are full-wave models that include impedance and phase velocity dispersion. I have successfully used both of these models to design microstrip filters at mm-wave frequencies [6].

All of our microstrip and stripline examples have been quite simple (OK, trivial) with well-known solutions. Nevertheless, these simple problems are quite useful for getting comfortable with a new software tool before we attempt a more complicated problem.

VIII. CPW with Dielectric Overlay

In multilayer PC boards, we can dream up many transmission line configurations that are not addressed by standard analytical models. If we can compute an impedance and phase velocity for the structure, we can include these non-standard cross-sections in our circuit designs.

One afternoon, a co-worker brought me a sketch of the geometry shown in Fig. 10. He wanted to use this CPW like structure with dielectric overlay to route some signals on his board.
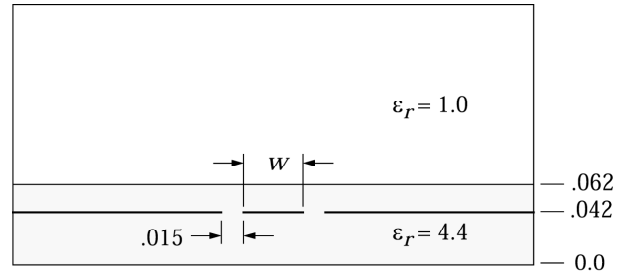

Fig. 10.   Geometry and dimensions (inch) for CPW with dielectric overlay example.

We used *QuickField* to compute impedance as a function of the center line width, w. In this case we chose to ignore metal thickness. Fig. 11 shows the mesh we developed for this problem. The mesh was fine-tuned at the edges of the strips to maximize accuracy.
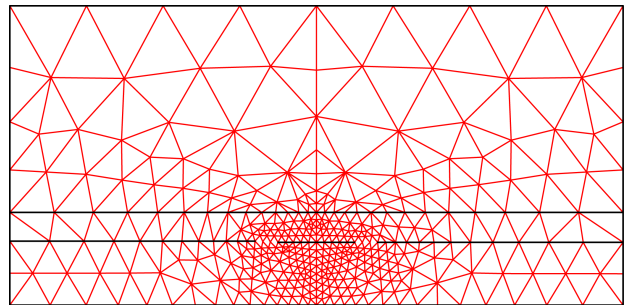

Fig. 11.   FEM mesh for the CPW with overlay example. The mesh was fine-tuned at the edges of the strips to maximize accuracy.

After setting up our problem and doing the first solution, we can look at the lines of constant potential to make sure they make sense, Fig. 12. The center strip is set to 1.0 volt and the contour lines have an increment of 0.1 volt.
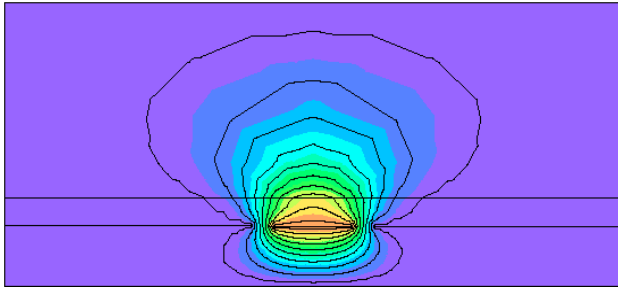
Fig. 12. Lines and shaded contours of constant potential. The center strip is at 1V and the contour lines have an increment of 0.1V.

We computed results for several line widths while we held the gap constant at 0.015 inch. The results are tabulated in Table II.

TABLE II
CPW WITH OVERLAY RESULTS

w (in.)	C (F/m)	C_o (F/m)	Z_o (ohms)	ε_{eff}
.030	1.376e-10	3.373e-11	49.26	4.08
.040	1.490e-10	3.720e-11	45.07	4.01
.050	1.583e-10	3.991e-11	42.22	3.97
.060	1.711e-10	4.333e-11	38.97	3.95

Once we have the impedance and effective dielectric constant information, we can plug those into an ideal transmission line element in our favorite linear simulator. With a little more work, we could fit curves to the Z_o and ε_{eff} data and program those equations into the linear simulator using the equation block feature.

IX. Buried Transmission Lines

In multilayer PC boards, we often use buried transmission lines to route RF signals. To increase the isolation between lines in the same layer, we sometimes bring metal close to the signal line and connect it to the ground planes above and below with vias. Depending on the relative dimensions, we might call this CPW or we might call it stripline. The label we put on it is less important than our ability to analyze any structure than can be manufactured at a reasonable cost. A typical cross-section is shown in Fig. 13.
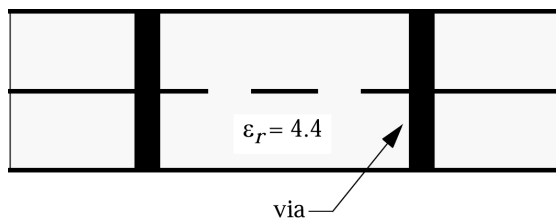

Fig. 13. Typical buried, shielded transmission line. Only three layers of a multilayer board are shown.

This may represent three metal layers out of a board with eight layers or more. There may or may not be vias close to the edge of what we intend to be buried ground planes. The thickness of the dielectric layers might be anywhere from 0.005 inch to 0.032 inch. In addition, the layers may have different thickness.

Fig. 14 is a cross-section from an actual eight layer PC board, only three of the metal layers are shown. We can clearly see the trapezoidal cross-section of the etched conductors. Also, note that the distance to the upper and lower ground planes is not equal.
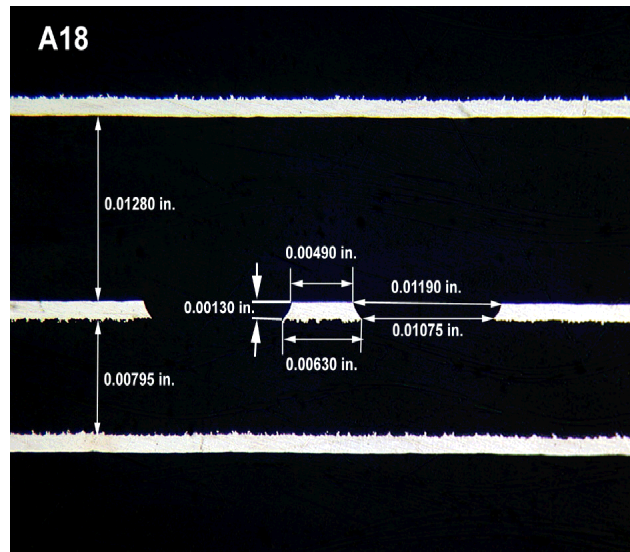

Photo courtesy of M/A-COM

Fig. 14. Actual cross-section from an eight layer PC board, only three of the metal layers are shown.

Using *FlexPDE,* we can easily compute the impedance of this structure for the given dimensions. The finite element mesh for this problem is shown in Fig. 15.
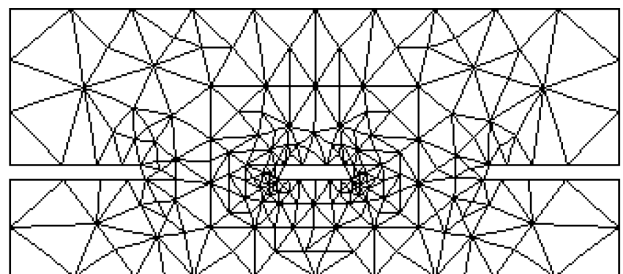

Fig. 15. The finite element mesh for the geometry shown in Fig. 14. The actual cross-sections of the conductors are included in the analysis.

We can also explore the effects of the trapezoidal cross-section on impedance. Alternatively, we might investigate the effects of the various ground planes on the computed

impedance. Once we are comfortable with these software tools, we are only limited by our imagination.

X. Conclusion

Although a lot of attention has been focused on the very capable and expensive 2.5D and 3D field-solvers, there are many useful problems that can be done easily and quickly with an inexpensive 2D cross-section solver. After a quick introduction to impedance calculation, we solved some basic microstrip and stripline problems. Anytime you are learning to use a new tool it is prudent to solve a few problems where the correct answer is known. Then you can move on to more interesting problems, like the CPW with overlay example and the shielded stripline example. This article was not intended to be a comprehensive review of these software packages, but rather a brief introduction to the concepts needed to calculate static transmission line impedances. Example files are available from the author via email.

There are some other applications of 2D PDE solvers that we did not have time to cover in this article. Several of my colleagues use these solvers to calculate capacitance for various active device geometries. One very sophisticated application is a combined parasitic and thermal analysis of a GaAs FET [7]. If you use complicated thin-film or thick-film resistor geometries, you can solve for the resistance and current distribution. We can also find the waveguide cutoff of a complex package cross-section with inhomogeneous, layered dielectrics. Perhaps we will cover some of these applications in a future note.

About the Author

Daniel G. Swanson, Jr. (d.swanson@ieee.org) is R&D Manager at Bartley R.F. Systems in Amesbury, MA. He designs filters for RF and microwave applications and is a consultant on applied electromagnetics to the RF and high speed digital communities. He also teaches a continuing education course, *RF & High Speed Digital Design Using Field-Solvers*, which is offered in the US and Europe. Mr. Swanson is a Fellow of the IEEE.

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[7] T. P. Budka, "Simultaneous Electrical and Thermal Modeling of GaAs FETs Using A Two-Dimensional Partial Differential Equation Solver, *Microwave Journal*, pp. 142-146, November 1997.

Appendix A – Software Vendors

FlexPDE
PDE Solutions
www.pdesolutions.com

QuickField
Tera Analysis Ltd.
www.quickfield.com

FEMLAB
COMSOL
www.femlab.com

Maxwell SI 2D, Serenade
Ansoft Corp.
www.ansoft.com

LINPAR and *MULTLIN*
Artech House Publishers
www.artech-house.com

ELECTRO
Integrated Engineering Software
www.enginia.com

ElecNet
Infolytica Corp.
www.infolytica.com

Opera-2D
Vector Fields
www.vectorfields.com

ApsimRLGC
Applied Simulation Technology
www.apsimtech.com

LINMIC+/N
AC Microwave
www.linmic.com

ADS, LineCalc
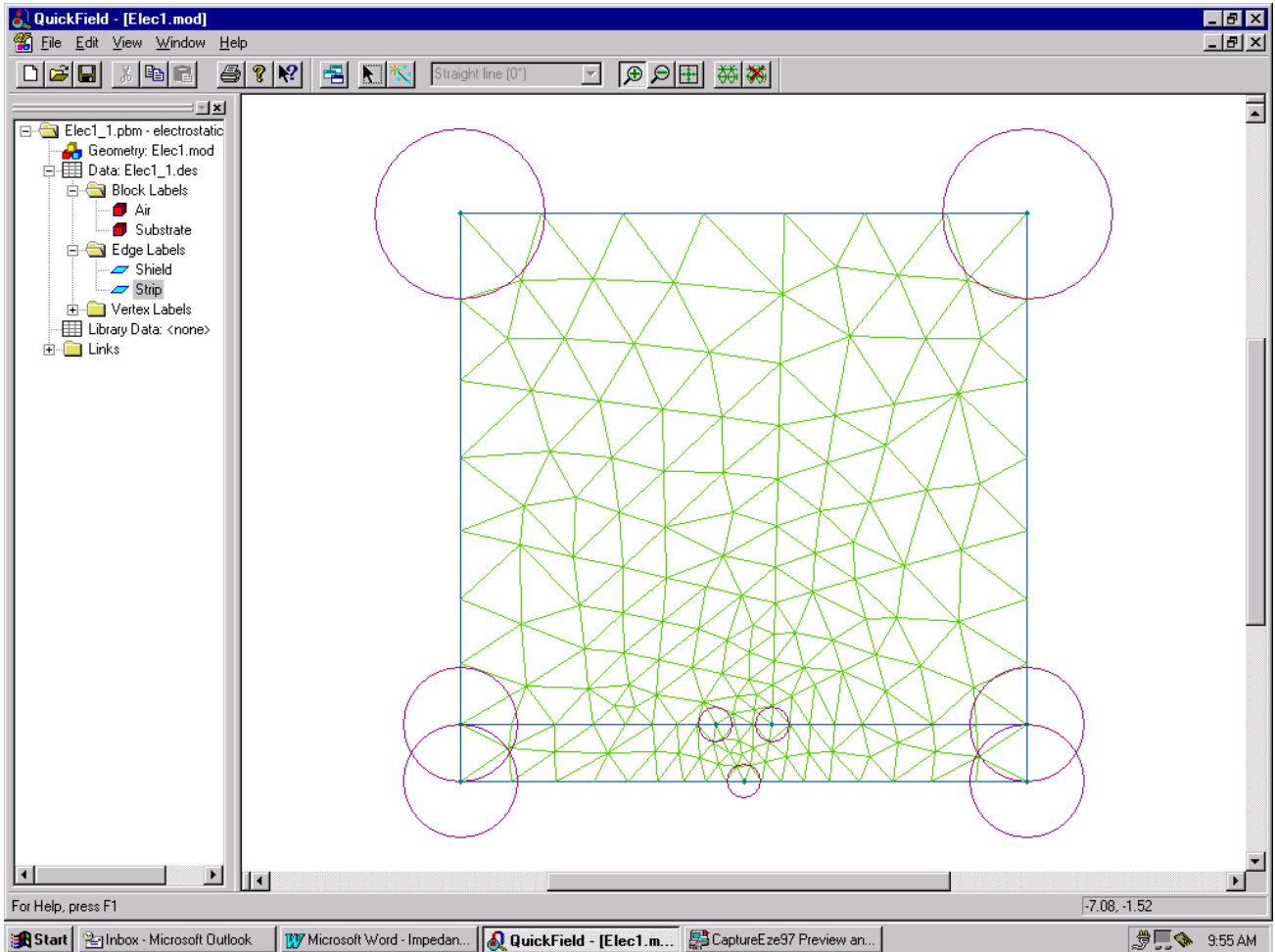Agilent Eesof EDA
http://contact.tm.agilent.com/tmo/hpeesof

Fig. 3.   A screen capture from *QuickField* showing the geometry of a simple microstrip in a box problem. At selected vertices, the user defines the radius of circle that controls the mesh size at that point. There is a "dummy" vertex at the bottom, center that forces extra mesh under the strip.
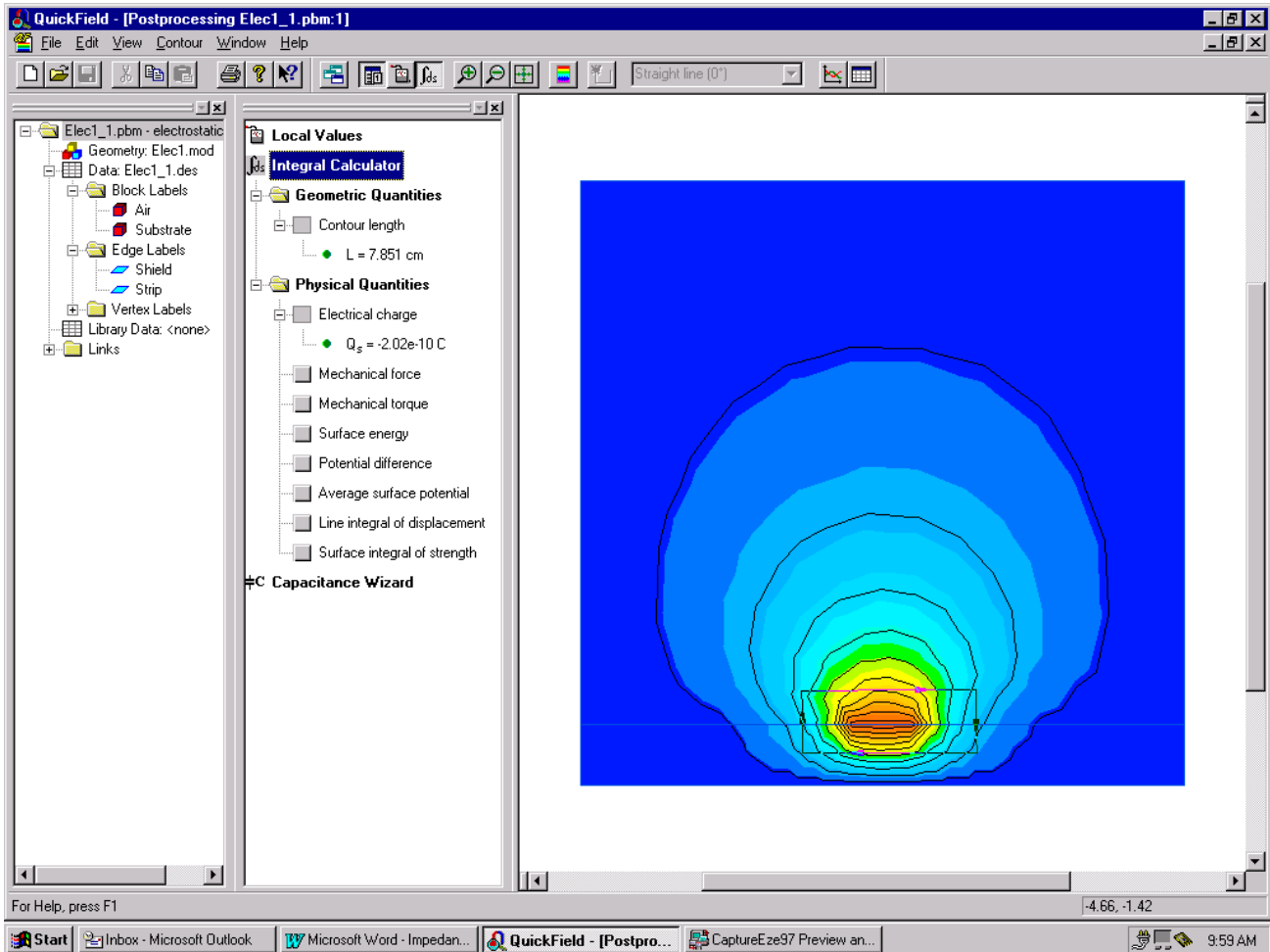
Fig. 4.   An example of post-processing using *QuickField*. The plot includes constant potential contours, shaded potential contours and the closed integration path for computing the total charge on the strip.
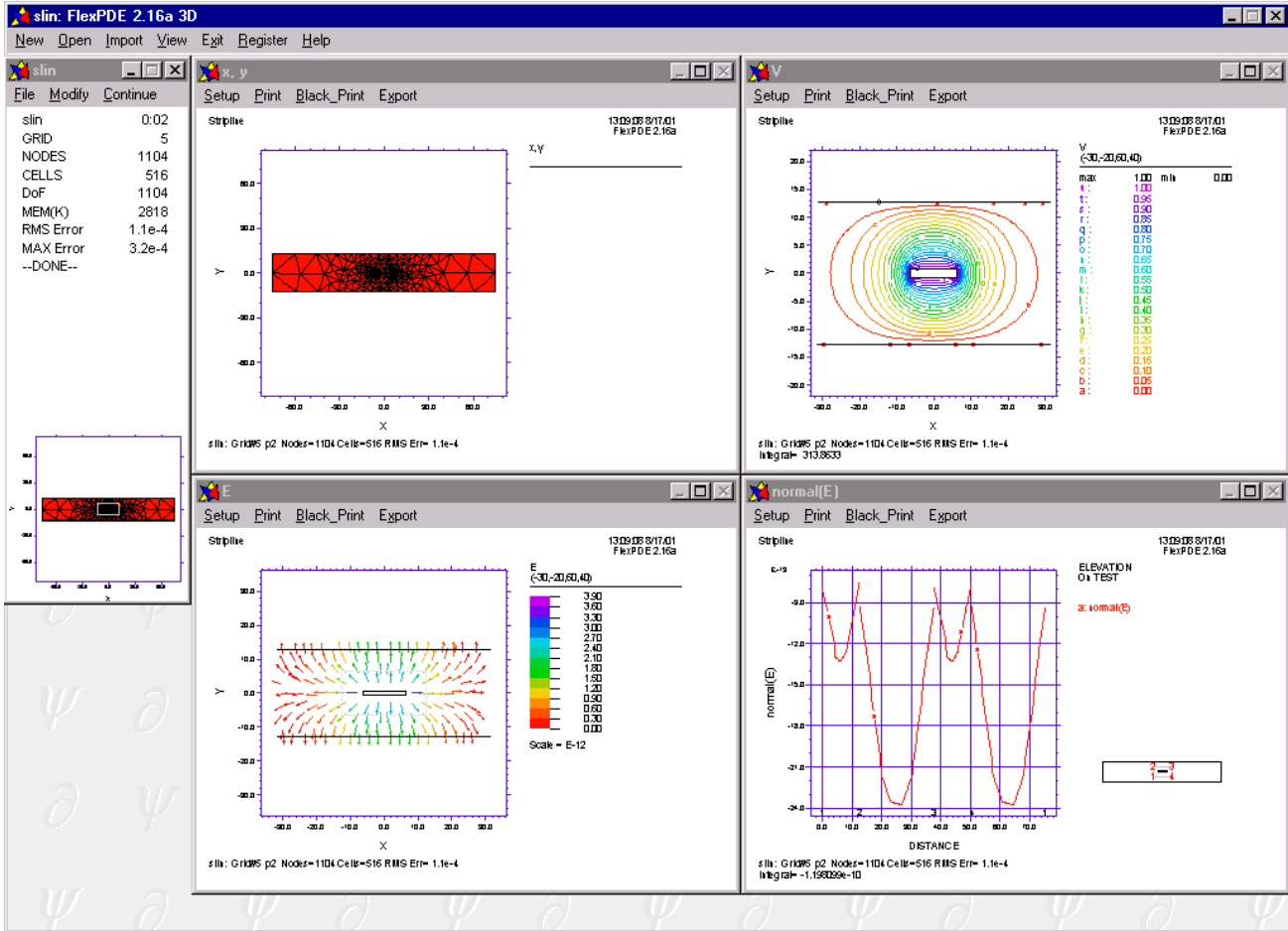
Fig. 6.   Post-processing the stripline results in *FlexPDE*. Starting at the upper left and going clockwise the graphs are; the finite element mesh, contours of constant potential, the normal component of the E-field along the integration path and a vector E-field plot.