Application the theory of similarity at simulation of wide-scaling objects

[Methods of scaling for QuickField simulation]

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Dynamic processes and their equations in immobile media

1. **Equation of the mass transfer,**

stipulated by diffusion in result of non-uniform concentration of particle (self-diffusion) at the absence of thermo-diffusion and baro-diffusion:

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$$\frac{\partial u}{\partial t} - D_1 \nabla^2 u = 0$$

Here $u \equiv \rho$ is a density of medium $\left[\frac{kg}{m^3}\right]$, D_1 is a diffusion coefficient $\left[\frac{m^2}{\sec}\right]$,

$\nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$ is the Hamilton's operator (nabla),

$\nabla^2 = \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the Laplace's operator.

In the gaseous medium $D_1 = (1/3)\langle v \rangle \langle l \rangle$, where

$\langle v \rangle$ is the mean mass velocity of the medium particles,

$\langle l \rangle$ is the average length of free path for the particles of medium.

2. **Equation of the heat transfer.**

$$c_p \rho \frac{\partial u}{\partial t} - \nabla \cdot (\lambda \nabla u) = 0$$

Here $u \equiv T$ is a temperature of medium;

λ is a coefficient of heat conductance, $\left[\frac{Joule}{m \cdot \sec \cdot \cdot ^\circ K}\right]$;

c_P is the isobaric heat capacity, $\left[\frac{Joule}{kg \cdot ^\circ K}\right]$; ρ is a density of medium, $\left[\frac{kg}{m^3}\right]$.

3. **Equation of magnetic field diffusion.**

$$\frac{\partial u}{\partial t} - \frac{1}{\mu\sigma} \nabla^2 u = 0$$

Here $u \equiv B$ is the magnetic field induction, $Tesla$;

μ is a magnetic permeability of medium $\left[\frac{Henry}{m}\right]$;

σ is a coefficient of electrical conductivity of medium $\left[\frac{1}{Ohm \cdot m}\right]$.

For the homogeneous and isotropic media all these equations can be written in the common form of diffusion equation with corresponding meaning of diffusion coefficient for each case.

$$\frac{\partial u}{\partial t} - D_K \nabla^2 u = 0$$

1. For the **equation of the mass transfer** a coefficient of particle diffusion must be used :

$$D_K = D_1 = \frac{1}{3} < v > \cdot < l > \quad \left[\frac{m^2}{\sec}\right]$$

2. For the **equation of the heat transfer** a coefficient of the heat diffusion or, by other words, coefficient of temperature conductance for medium must be used :

$$D_K = D_2 = \frac{\lambda}{c_P \cdot \rho} \quad \left[\frac{m^2}{\sec}\right]$$

3. For the **equation of magnetic field diffusion** a coefficient of the magnetic field diffusion must be used :

$$D_K = D_3 = \frac{1}{\mu \cdot \sigma} \quad \left[\frac{m^2}{\sec}\right]$$

Why it is of interest for us to find the correspondence, one-to-one just up to similarity, between the solutions of diffusion equation for different object of simulation, with great difference in dimensions of model and time duration of process?

In the practice of simulation a developer can deal with a needs of modeling of diffusion process as in the big systems of giant dimensions at very long duration of process, from one side, and in the microscopic systems with extremely short time of process, from another side.

Not always it is reasonable to perform a simulation of process in computer in the real scale of object dimensions and, what is especially essential, in the real time. The principle of the solutions simulation, which will be considered now, gives a possibility to transform the solution, obtained for some fixed time-space characteristics, into the solution for geometrically similar model with time-space characteristics of another range.

WHAT OBJECTS CAN YOU MEET IN YOUR PRACTICE OF DYNAMIC PROCESS SIMULATION WITH QUICKFIELD?

The program **QuickField** has the wide range of allowable limits for dimension and time borders of simulation model:

Since 1 μm up to 1 km – in dimensions.

Since 10^{-32} sec up to very long processes - in time.

The specific property and positive advantage of the QuickField is a possibility to build the model geometry in the graphical editor by simple drawing.

The space frame of model in computer must be taken in definite correspondence with a scale of objects either sub-micro dimensions or giant dimensions.

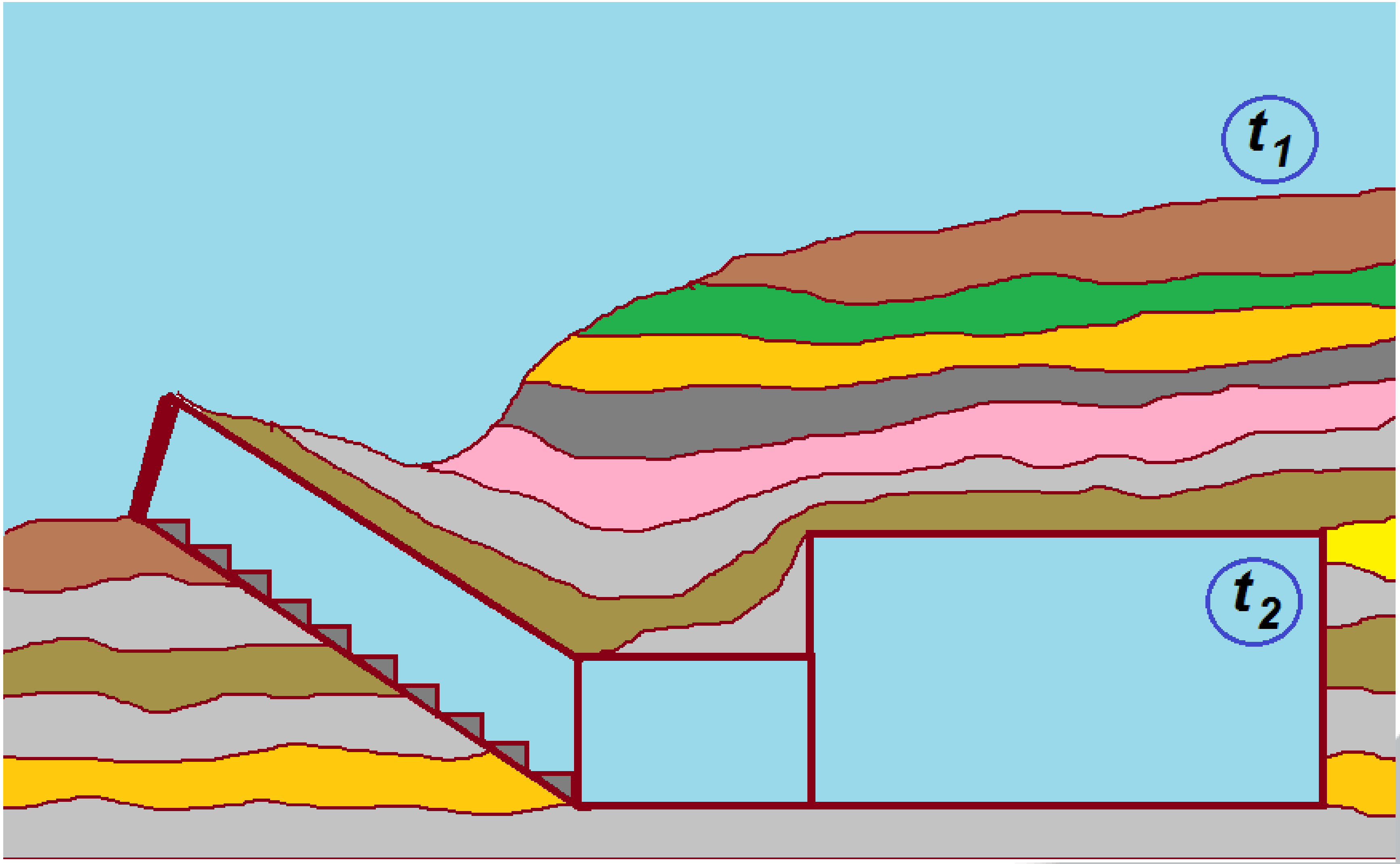
I. Let we have a long process in the object of large enough dimension.

For example, the process of the heat diffusion across the multi-layer thermal protection of under-ground fruit-store can have duration in real time near ½ year (Figure in the next slide).

It does not mean that we must be sitting near computer 6 month to get the result of the heat diffusion simulation.

We must use the scaling factors to provide the similarity between the process in our simulation model and a real process at the reasonable time of problem solution.

Heat penetration across the multi-layer cover – example of thermal diffusion during a long time at big enough dimensions of object.



AND WHAT ABOUT SIMULATION OF THE FIELD PROCESS IN THE MICROSCOPIC OBJECTS?

In this case, we must be sure that the model of very small micro-object can be drawn successfully in the frame of graphical editor of program. Only so the advantages of the model drawing in the graphical editor of program can be applied usefully.

It means that the dimensions of real object must be reduced or increased by special method to reach always a correspondence with allowable dimensions of model in the working field of graphical editor of program.

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Beside of correlation between real dimensions of object and its dimensions in the simulation model, the question appears about time duration of process in model in comparison with duration of real process in object of simulation.

II. Example of microscopic objects simulation.

The necessity of micro-objects simulation appears often at the study of micro-magnetic process as well as the study of thermal processes at the micro-miniaturization of elements of computer techniques and scientific instruments.

A simulation of thermal or electromagnetic process in the objects of sub-micro dimensions can be applied effectively also at development of new materials and wares of nano-technologies.

As the example of the electric field calculation in the microscopic object (near 50 nm) we can consider calculation of the oscillating field of micromagnetic dipole described in the journal *IEEE Transactions on Magnetics* (next slide).

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[1] E.Martinez, L.Torres and L.Lopez-Diaz. Computing of solenoidal field in micromagnetic simulations*, IEEE Transactions on Magnetics*, vol. 40, no.5, Sept. 2004, pp. 3240 – 3243.

At the investigation of micromagnetic processes it was calculated electric field in the volume of cube with side 50 nm (published by E.Martinez, L.Torres and L.Lopez-Diaz, 2004 **[1]**)

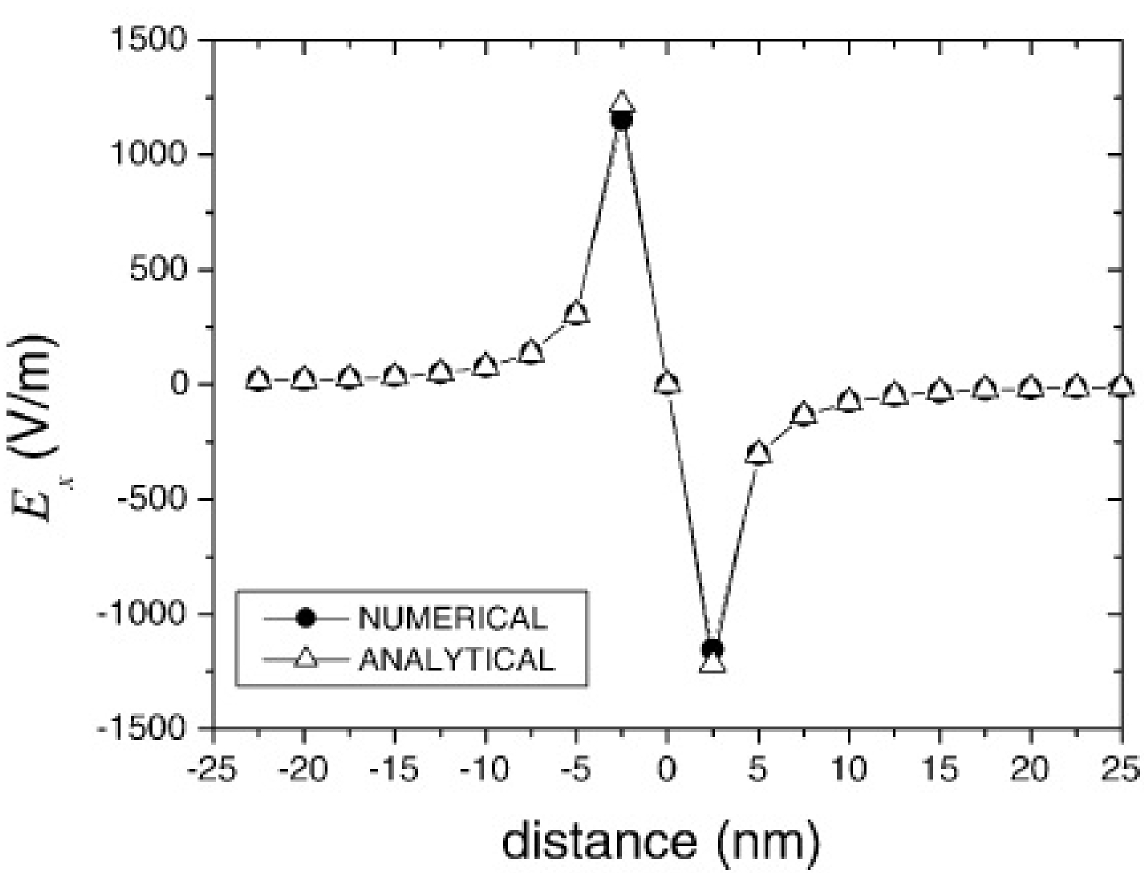

Figure 3 of **[1]**:

Electrical field E_x of oscillating dipole as a function of the distance to the dipole along *y* axis. The instant of time is

$t = 1.32 \cdot 10^{-32}$ sec.

Analytical solution in: J.D.Jackson, Classical Electrodynamics, New York, Wiley, 1998.

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Comparison of analytical and numerical solutions.

The most suitable processes for discussion of the model scaling at simulation of dynamic phenomena is the next processes:

1) transient heat penetration into medium with linear properties - ***Diffusion of the Heat***;

2) transient electromagnetic field penetration into ferromagnetic medium which has electrical conductivity – process called as ***Diffusion of the Electromagnetic Field.***

There is possible to show here the mutual interconnections of time scale with dimensions scale, ***from one side***, and with thermo-physical or electromagnetic parameters of medium (as conductance for temperature (1) or magnetic permeability and electrical conductivity (2)), ***from another side.***

APPLICATION OF THE FIELD SIMILARITY PRINCIPLE FOR TWO MODELS OF FERROMAGNETIC MEDIUM AT DIFFERENT DIMENSIONS AND DIFFERENT ELECTROMAGNETIC PROPERTIES (PERMEABILITY *μ* AND CONDUCTIVITY *σ*).

The electromagnetic process for each of two samples under consideration can be described by typical equation of non-stationary diffusion

$$\frac{\partial B}{\partial t} = D \Delta \vec{B} \; ; \qquad D = \frac{1}{\mu \sigma}$$

where B is the local value magnetic induction, D is the local meaning of the field diffusion coefficient.

We shall imply for the 1st sample values μ_1, σ_1, and for 2nd sample values μ_2, σ_2.

The condition of the field distribution similarity can be especially easy established when both samples under consideration present the uniform media with linear properties.

The conditions of the field diffusion similarity can be introduced on the base of one of two next approaches:

- **1.** When the coefficient of the field diffusion has the same value for both samples: $D_1 = D_2$.
- **2.** When a correlation between duration of electromagnetic process in the samples (T_1 / T_2) is given ahead with no connection with ratio of characteristic dimensions (X_1 / X_2) of one and another sample.
- **3.** In general case of equality for normalized meanings of diffusion coefficients: $D_1^* = D_2^*$. These normalized meaning will be considered in the following pages.

APPROACH 1. For application of the conditions of the field similarity it is necessary to introduce the characteristic dimension and time for each sample as the basic parameters of corresponding processes:

$$T_1 = T_{1bas}, \quad X_1 = X_{1bas}, \quad B_1 = B_{1bas}$$

for the 1st sample;

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$$T_2 = T_{2bas}, \quad X_2 = X_{2bas}, \quad B_2 = B_{2bas}$$

for the 2nd sample.

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Equation of the field diffusion now can be re-written in normalized variables $t^* = t / T_{bas}$; $B^* = B/B_{bas}$;

$x^* = x / X_{bas}$; $y^* = y / X_{bas}$; $z^* = z / X_{bas}$.

(for each sample with indexes 1, 2 respectively).

Equation of the field diffusion now is looking so:

(let index 1 belongs to real object, and index 2 belongs to simulation model). :

$$\partial B_1^* / \partial t_1^* = D_1 \frac{T_{1bas}}{X_{1bas}^2} \Delta_1^* B_1^*; \quad (1) \qquad \partial B_2^* / \partial t_2^* = D_2 \frac{T_{2bas}}{X_{2bas}^2} \Delta_2^* B_2^*. \quad (2)$$

Here $\Delta_1^*, \ \Delta_2^*$ are the Laplace operators in non-dimensional variables,

$$B_1^* = f_1(x_1^*, y_1^*, z_1^*, t_1^*); \qquad B_2^* = f_2(x_2^*, y_2^*, z_2^*, t_2^*)$$

are the normalized functions of the field values depending on the non-dimensional coordinates and time.

$B_1^*, \ B_2^*$ can be presented in any suitable scale as developer wanted.

There is of interest for us to consider the final field distribution at the time moment $t_{comp} = T_{2bas}$, when the time t_{comp} of problem solution at the computer can reach a full time of diffusion in the simulation model.

At the equality of normalized meanings of diffusion coefficient

$$D_1^* = D_2^*, \quad \text{or} \quad D_1 \frac{T_{1bas}}{X_{1bas}^2} = D_2 \frac{T_{2bas}}{X_{2bas}^2} \qquad (3)$$

the equation (1) is identical with equation (2).

It means that non-dimensional solution of equation (2) coincides with non-dimensional solution of equation (1), i.e.

$$f_1(x_1^*, y_1^*, z_1^*, t_1^*) = f_2(x_2^*, y_2^*, z_2^*, t_2^*)$$

So, the normalized values of diffusion coefficients are **the general criteria of similarity** for equations (1) and (2) :

$$D_1^* = D_1 \frac{T_{1bas}}{X_{1bas}^2}; \quad D_2^* = D_2 \frac{T_{2bas}}{X_{2bas}^2}.$$

If we want to use the 1st approach to similarity, i.e. to keep $D_1 = D_2$,

for the similarity of the field distribution at the final time instant we must put the equality

$$T_{1bas} / T_{2bas} = (X_{1bas} / X_{2bas})^2 .$$

APPROACH 2. The second approach is based on the idea of accepted ***a priori*** the equality of the field diffusion velocity into the massive of both samples material:

$$V_{1\,diff} = V_{2\,diff} .$$

The field diffusion velocity can be defined via coefficient of diffusion $\boldsymbol{D}$ and a skin-depth $\boldsymbol{\delta}$ for the field penetration: $V_{diff} = D/\delta = 1/(\mu \cdot \sigma \cdot \delta)$.

General definition of the field skin-depth $\delta = D/V_{diff}$ gives a possibility to consider the instantaneous depth of the field penetration as linear function on time (what supposes the linear properties of medium):

$\delta(t) = V_{diff} \cdot t.$ Respectively, $V_{diff}(t) = 1/(\mu \cdot \sigma \cdot \delta(\tau))$

Jointly with a this general definition of the velocity of diffusion we have a well known definition for the depth of field penetration:

$$\delta^2(t) = D \cdot t; \quad \delta(t) = \sqrt{\frac{t}{\mu\sigma}} .$$

From the accepted ***a priori*** equality of the field diffusion velocity for sample 1 and sample 2, if we take into account the correlation (3) for normalized diffusion coefficients as a needed condition for equation (1) and (2) identity, it follows the next correlation which must be kept for similarity of solution for the field at final time instant (t_{1max} or t_{2max}):

$$\frac{\delta_1(t_{\max})}{\delta_2(t_{\max})} = \frac{T_{2bas}}{T_{1bas}} \cdot \left(\frac{X_{1bas}}{X_{2bas}} \right)^2 . \qquad (4)$$

It is possible to take into attention that at the end of diffusion process the full depth of the field penetration has a trend to reach the characteristic dimension, i.e.

$$\left. \frac{\delta_1(t)}{\delta_2(t)} \right|_{t \to t_{\max}} \Rightarrow \frac{X_{1bas}}{X_{2bas}} . \qquad (5)$$

Using (5) , we have instead of (4) a new condition:

$$\boxed{\frac{T_{1bas}}{T_{2bas}} = \frac{X_{1bas}}{X_{2bas}}} .$$

When a correlation between duration of electro-magnetic process in the samples (T_{1bas} / T_{2bas}) is given ahead, we can define a ratio of dimensions.

In correspondence with APPROACH 2 the similarity of the final field distribution in result of the filed diffusion will be provided when the correlation between the characteristic dimensions of simulation model and real object will be taken proportionally to ratio of awaited duration of time solution in model and duration of diffusion in the real object:

$$\frac{X_{1bas}}{X_{2bas}} = \frac{T_{1bas}}{T_{2bas}}.$$

SOME DISCUSSION. The results of our consideration allow to conclude that application of the similarity principle according to **APPROACH 1** delivers absolutely formal identity of non-dimensional equation of diffusion for simulation model and for real object. Some violation of similarity in final distribution of field can be caused only by non-linearity of the material properties at samples under comparison.

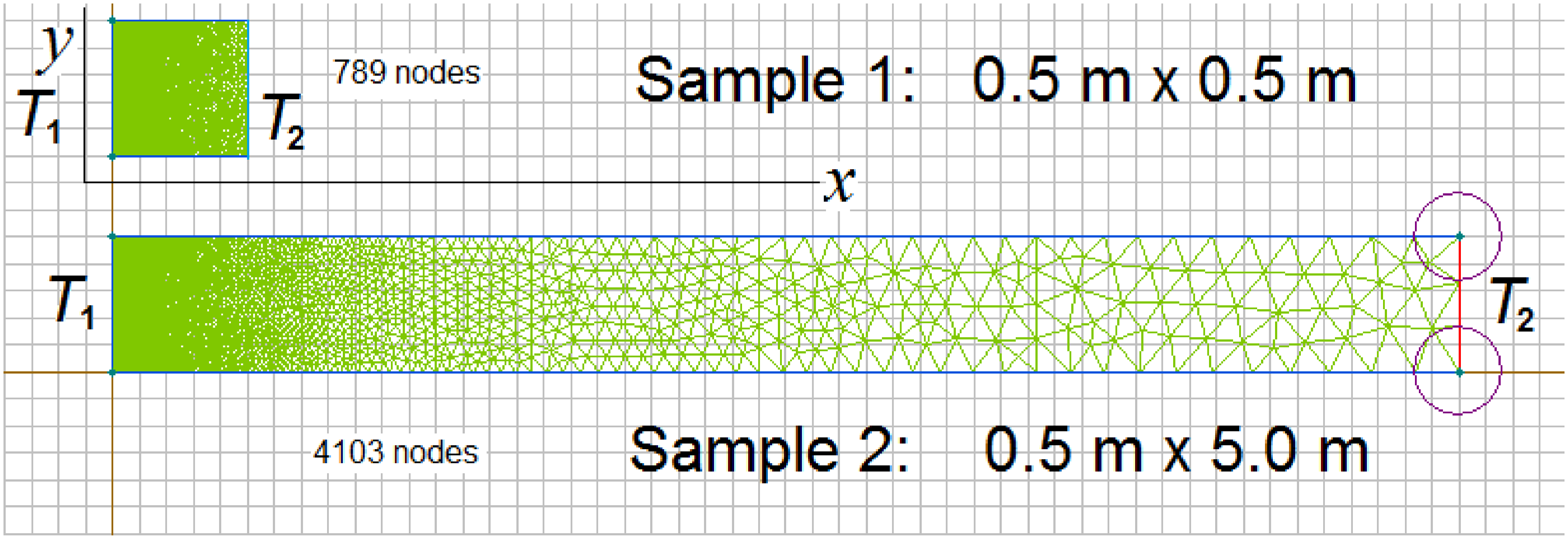
In general case APPROACH 1 can be applied at any meanings $D_1 \neq D_2$, because the formula (3) represents the general condition for identity of equation (1) and (2), i.e. the criteria of similarity at any combination of parameters $D,\ T_{bas},\ X_{bas}$.

At the same time, **APPROACH 2,** beside of formal condition for identity of diffusion equations for the real object and for its model in simulation, have used additional physical supposition about the equal velocity of the field diffusion into both samples. It leads to the proportionality between the characteristic dimension of sample and full time diffusion in this sample. This condition also can be violated at the non-linear properties of the samples materials.

As the rule, the magnetic permeability of magnetic materials depends on the level of magnetic induction. The program QuickField provides a possibility to input the magnetization curve into the simulation process. It means that the conditions of the field distribution similarity can be reached at the using of the same level of magnetic induction in the simulation model as in the real object. It can be provided by the proper choice of boundary conditions which define the magnetic flux (and respectively magnetic induction) in the cross section of simulation model.

Application of the principle of similarity for the solutions of equation of the **heat diffusion** will be demonstrated in quasi-one-dimensional consideration using models in the form of band of uniform material which are extracted from the area of arbitrary width along y-axis.

Temperature of material $T = f(x, t)$.



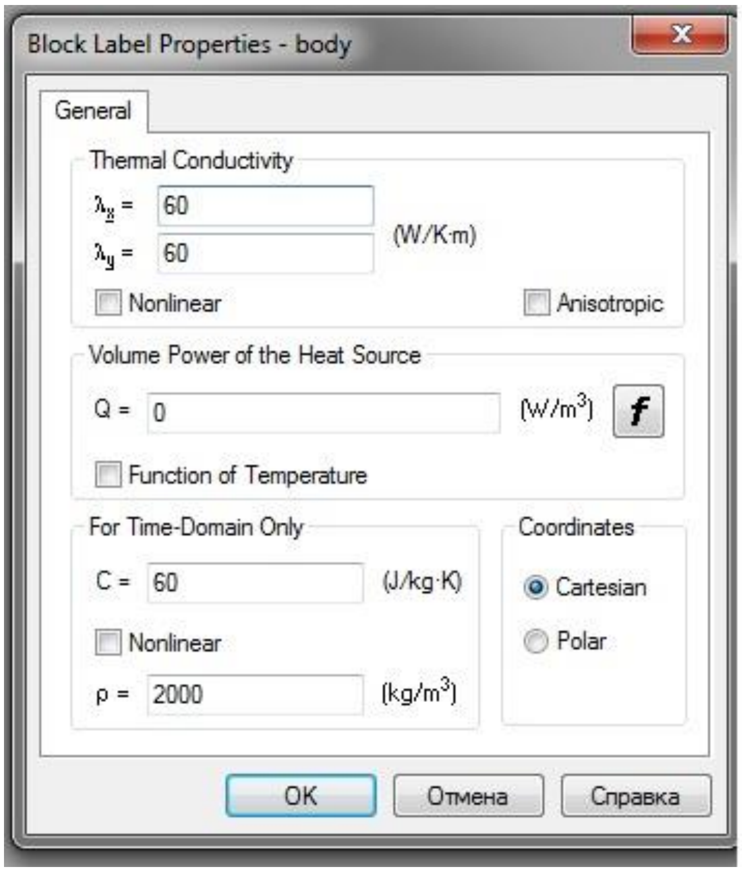
The solution of heat diffusion equation is obtained below for chosen parameters of medium at standard boundary conditions.

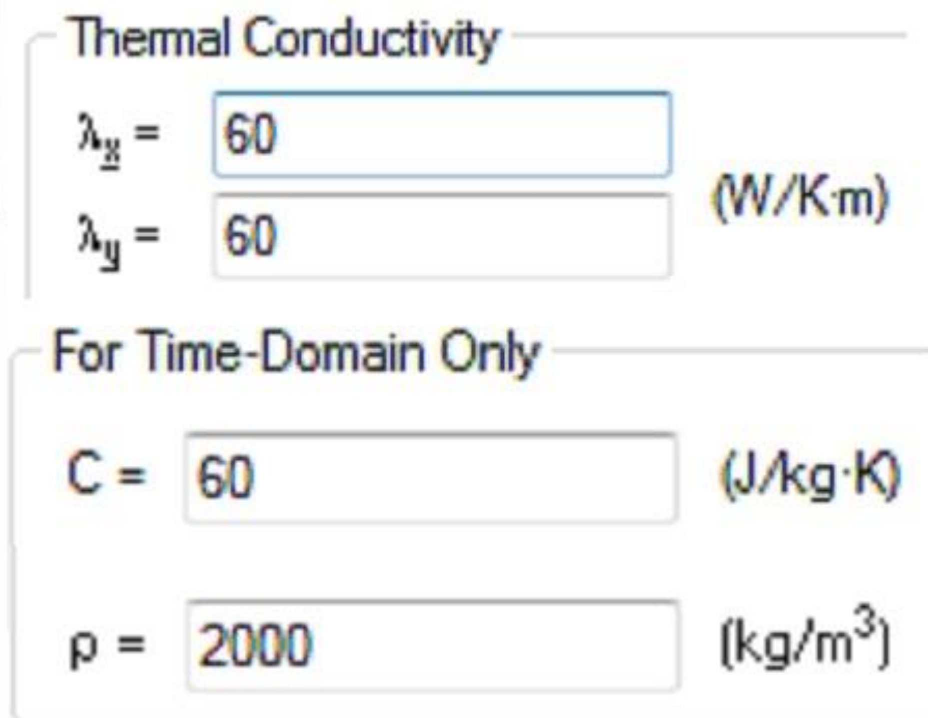
Blank for Body Properties in the **QuickField**.

Problem: **Transient Heat Transfer**.

Homogeneous isotropic body.

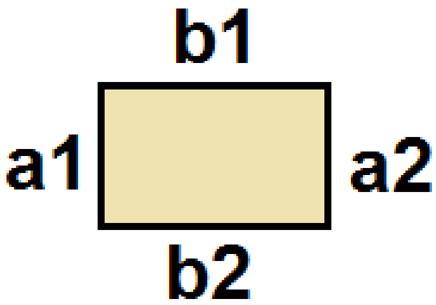
Coefficient of Heat Diffusion **$D = \lambda /(C\cdot\rho)$** [m^2/sec]





$D = \mathbf{60}(W/K\cdot m) / [\mathbf{60}(J/kg\cdot K)x\mathbf{2000}(kg/m^3)] =$

$= \mathbf{0.0005\ (m^2/sec)}$



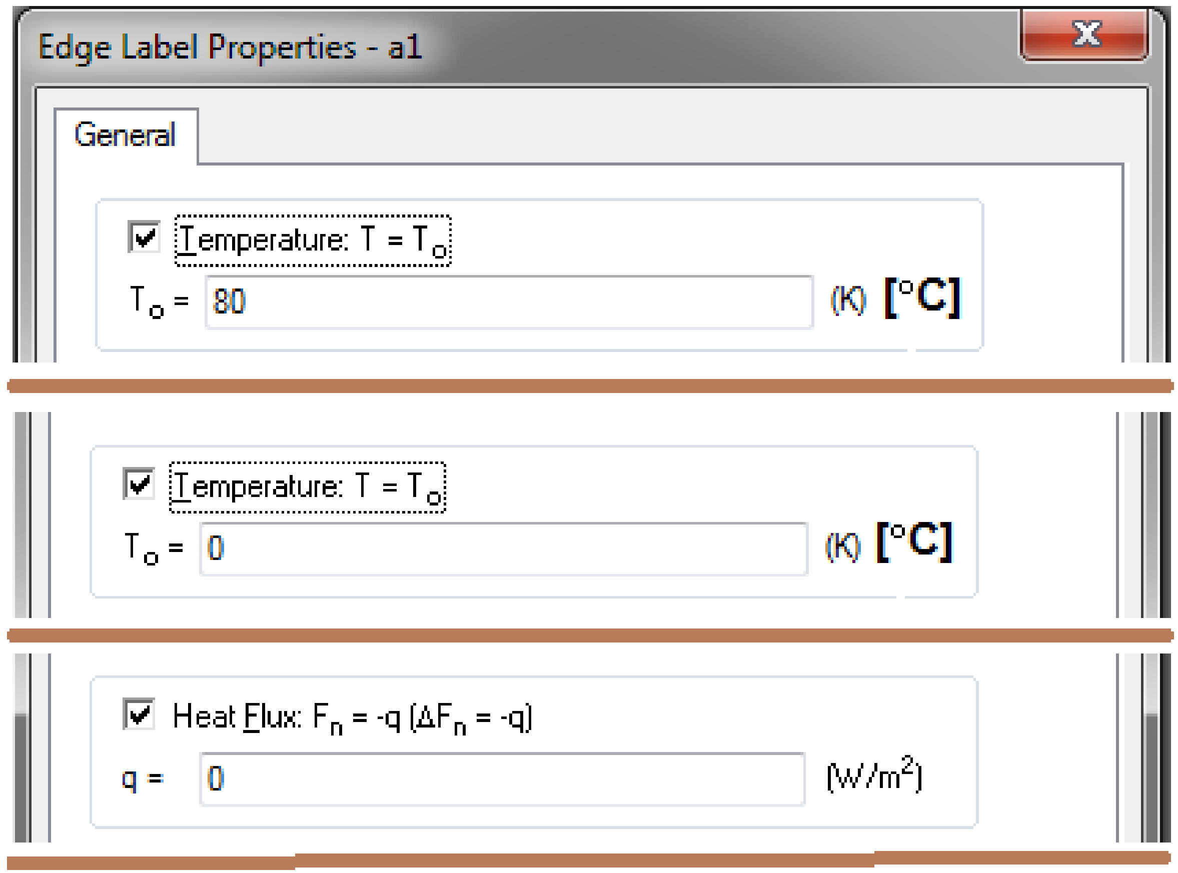
Boundary Conditions along the Edges.

Edge Label Properties
(blank in the **QuickField**).

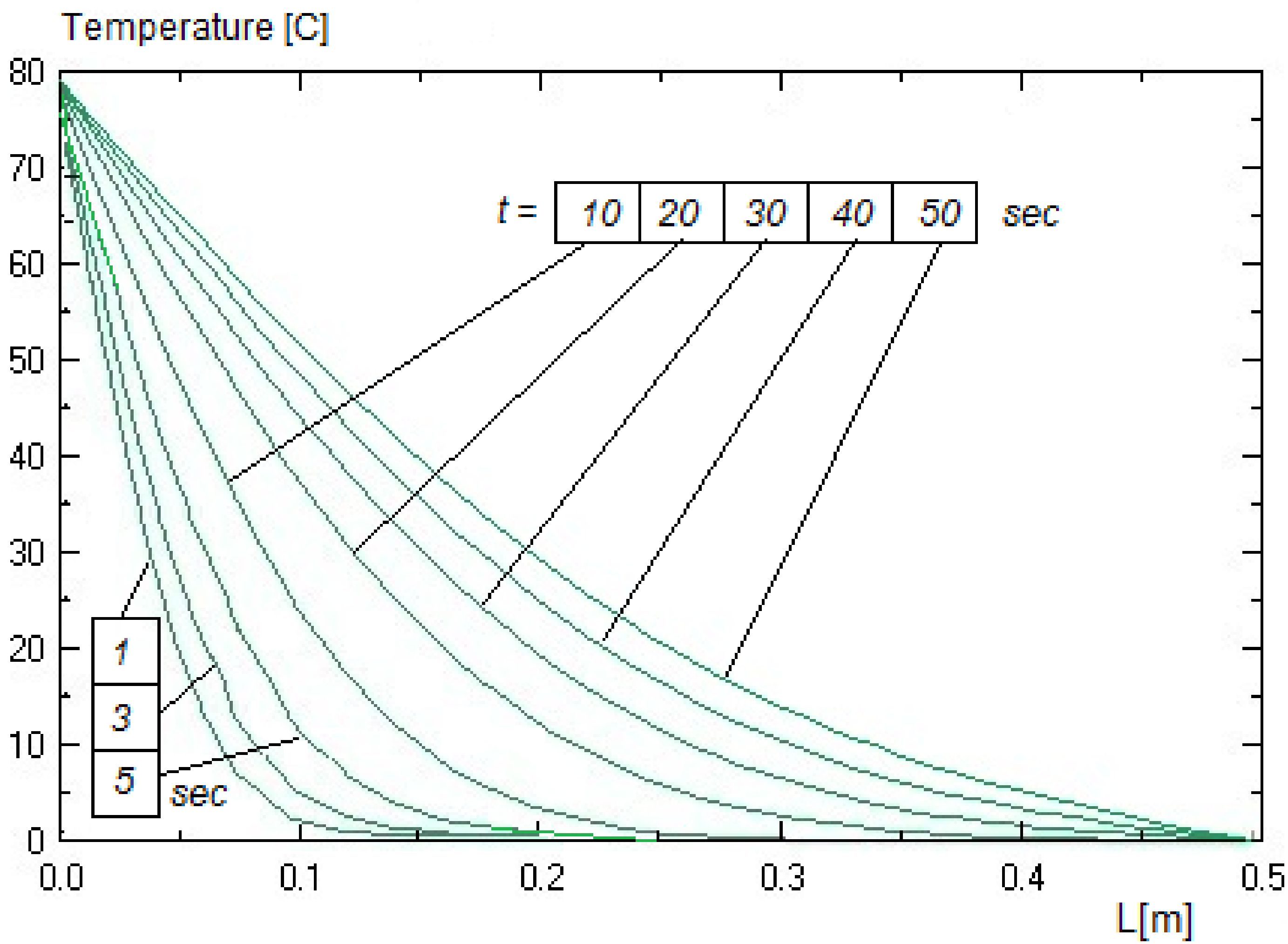
Along Label **a1**

Along Label **a2**

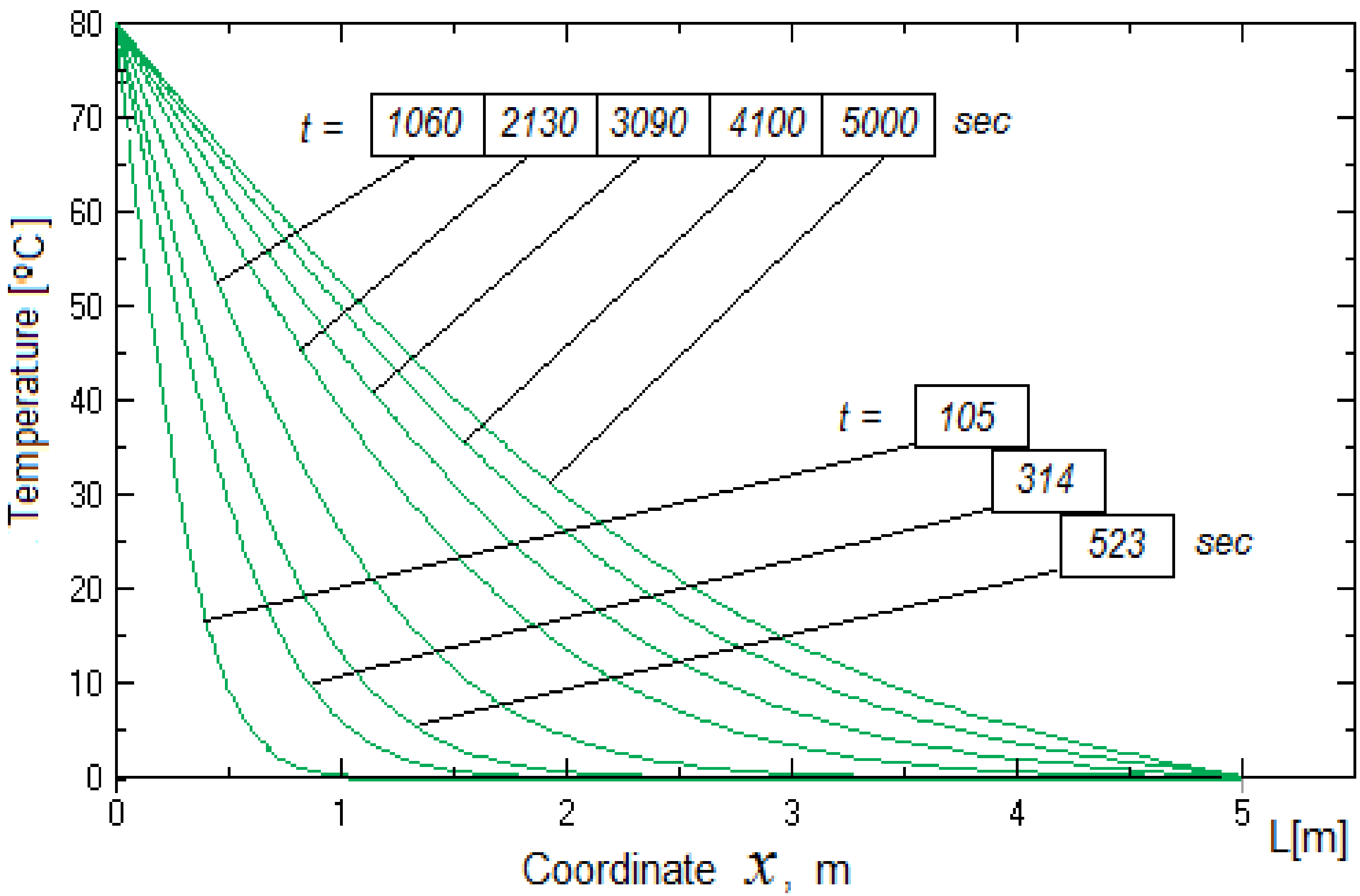
Along Labels **b1, b2**



Distribution of temperature in the space and time at $D = 0.0005\ m^2/sec$, $T_1 = 80°C$, $T_2 = 0°C$, $t_{MAX} = 50$ sec. The length of sample is L = 0.5 m.



The absolutely similar distribution of temperature in the space and time is obtained at the next parameters: L = 5.0 m, $t_{2MAX} / t_{1MAX} = 100$, $X_2 / X_1 = 10$, $D_1 = D_2 = 0.0005$ m^2/sec, saving the former boundary conditions. $t_{2MAX} = 5000$ sec.



Condition for the stationary state access

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Really, above at $D_1 = D_2$ $t_{1MAX} / t_{2MAX} = (X_1 / X_2)^2 = 100.$

Attention: the view of graphical dependence of temperature $T = f\,(x, t)$ not always gives a possibility to be sure in the identity of solutions on the base of similarity principle.

To be sure, it is useful to continue the both solutions up to the stationary state. As the factor of stationary mode achievement, the gradient of temperature can be used.

Heat flow is equal to $q = -\lambda \, grad \, T$.

If the gradient of temperature become constant along the coordinate x with high enough accuracy, it means the heat flow is stopped and respectively a temperature has reached a stationary distribution.

It allows to define the time of stationary state access t_{MAX} as the single-valued parameter.

Identity of boundary conditions as necessary condition for the similarity of solutions

At the variation of the sample dimension in coordination with a process duration the identity of boundary conditions presents the necessary condition for the similarity of solutions. It means that jointly with transformation of dimension and time we make a transformation of the heat flux along direction of diffusion under consideration:

$$q = -\lambda \, grad \, T$$

For one-dimension analysis $q = -\lambda \, \Delta T / \Delta x = -\lambda \, (T_1 - T_2)/\mathrm{L}$.

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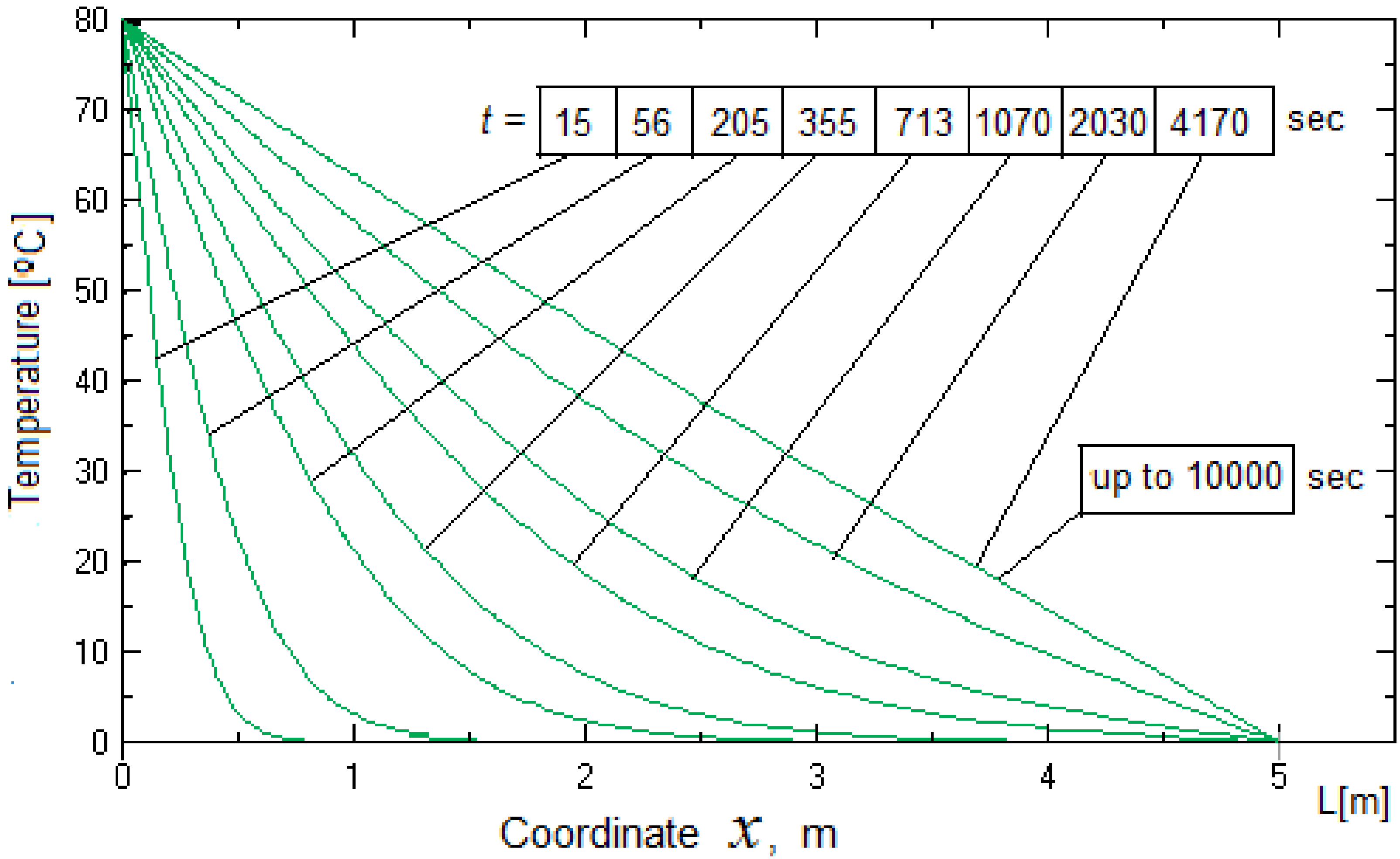
The special case of transient heat transfer is a problem with a given heat flux at the one or two borders of sample.

In this situation the identity of boundary conditions with respect to q also is a necessary condition for the similarity of solutions for scaled models at different dimensions and time of process.

Temperature distribution.

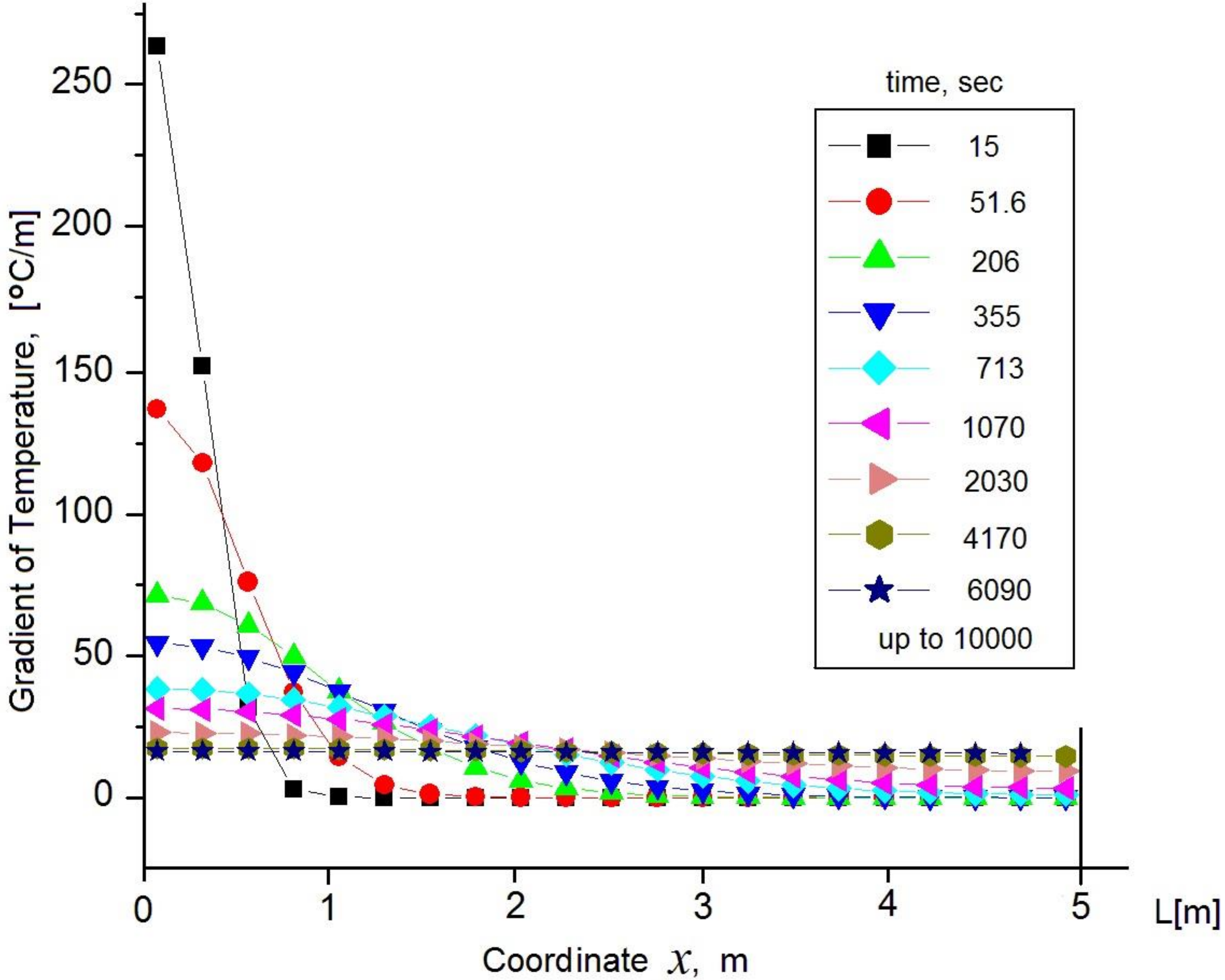
$D = 0.002\ m^2/sec$, $T_1 = 80°\ C$, $T_2 = 0°\ C$, $t_{MAX} = 10000$ sec.

Length of sample L = 5.0 m.



Gradient of Temperature distribution (L = 5 m).

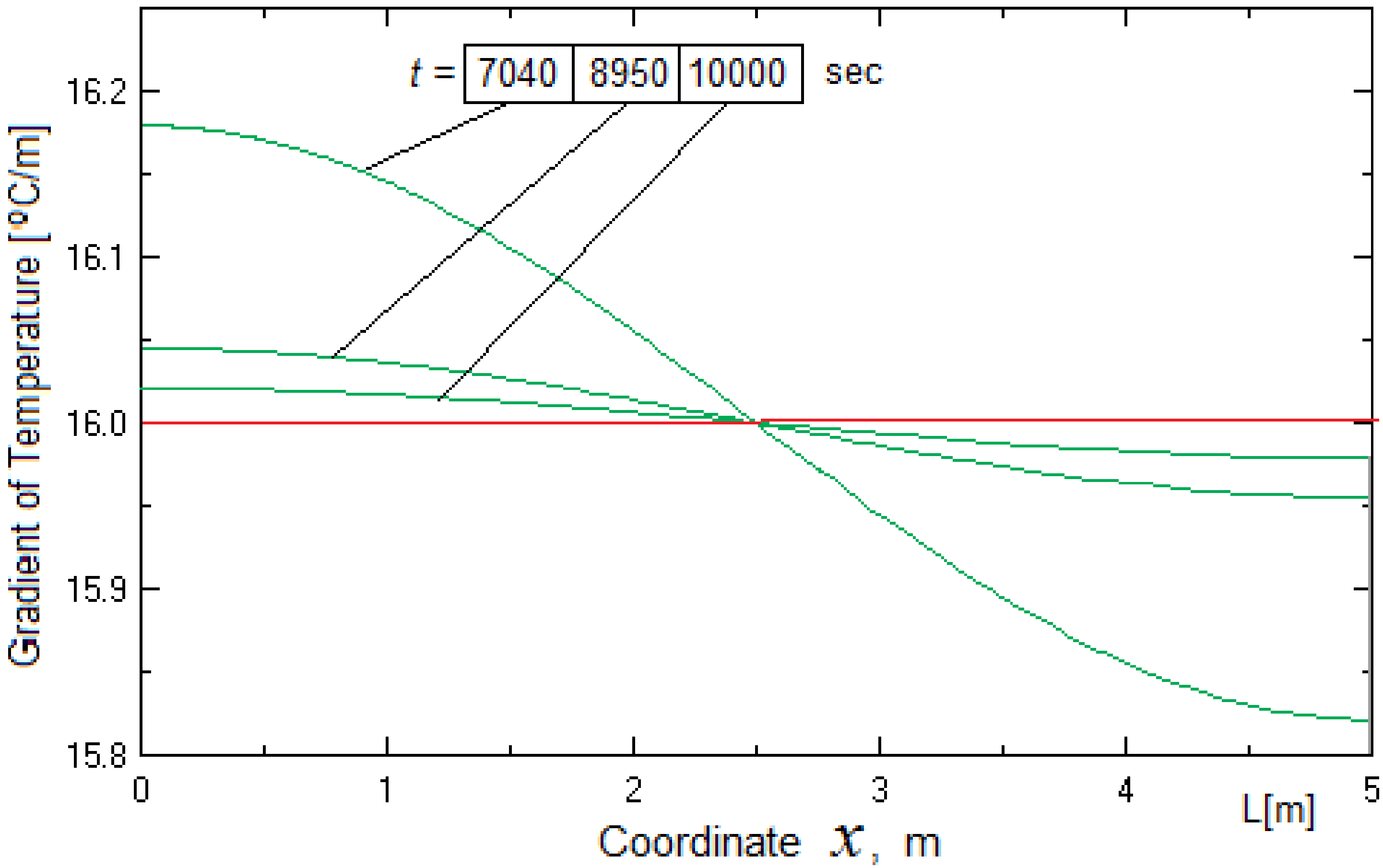
D = 0.002 m^2/sec, T_1 = 80°C, T_2 = 0°C, t_{MAX} = 10000 sec.



Gradient of Temperature at final instants of time (L = 5 m).

D = 0.002 m^2/sec, T_1 = 80°C, T_2 = 0°C, t_{MAX} = 10000 sec.

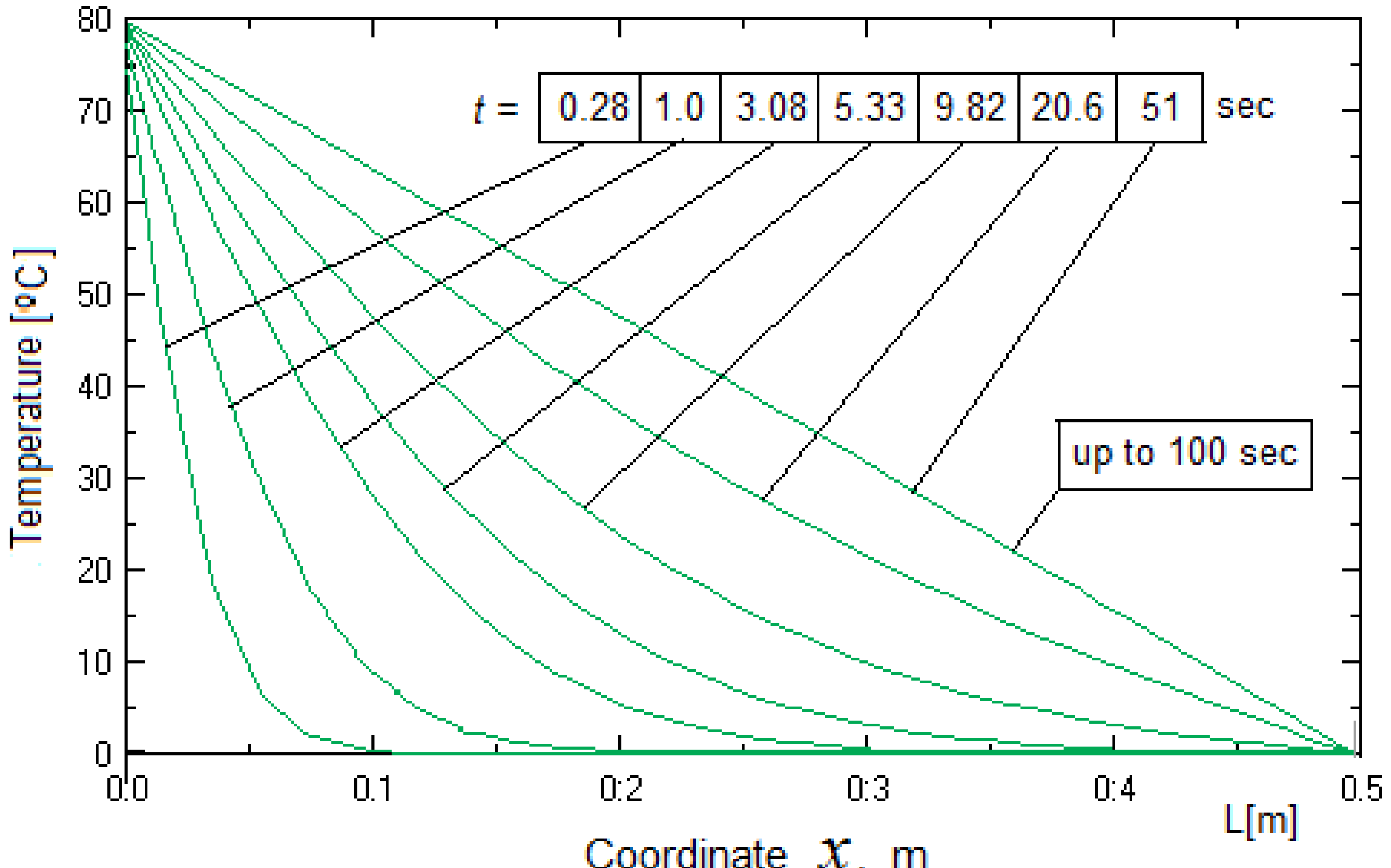
Final deviation of ***grad T*** from uniform value is less than 0.1%.



Temperature distribution for the scaled model (L = 0.5 m).

D = 0.002 m^2/sec. T_1 = 80°C, T_2 = 0°C, t_{MAX} = 100 sec.

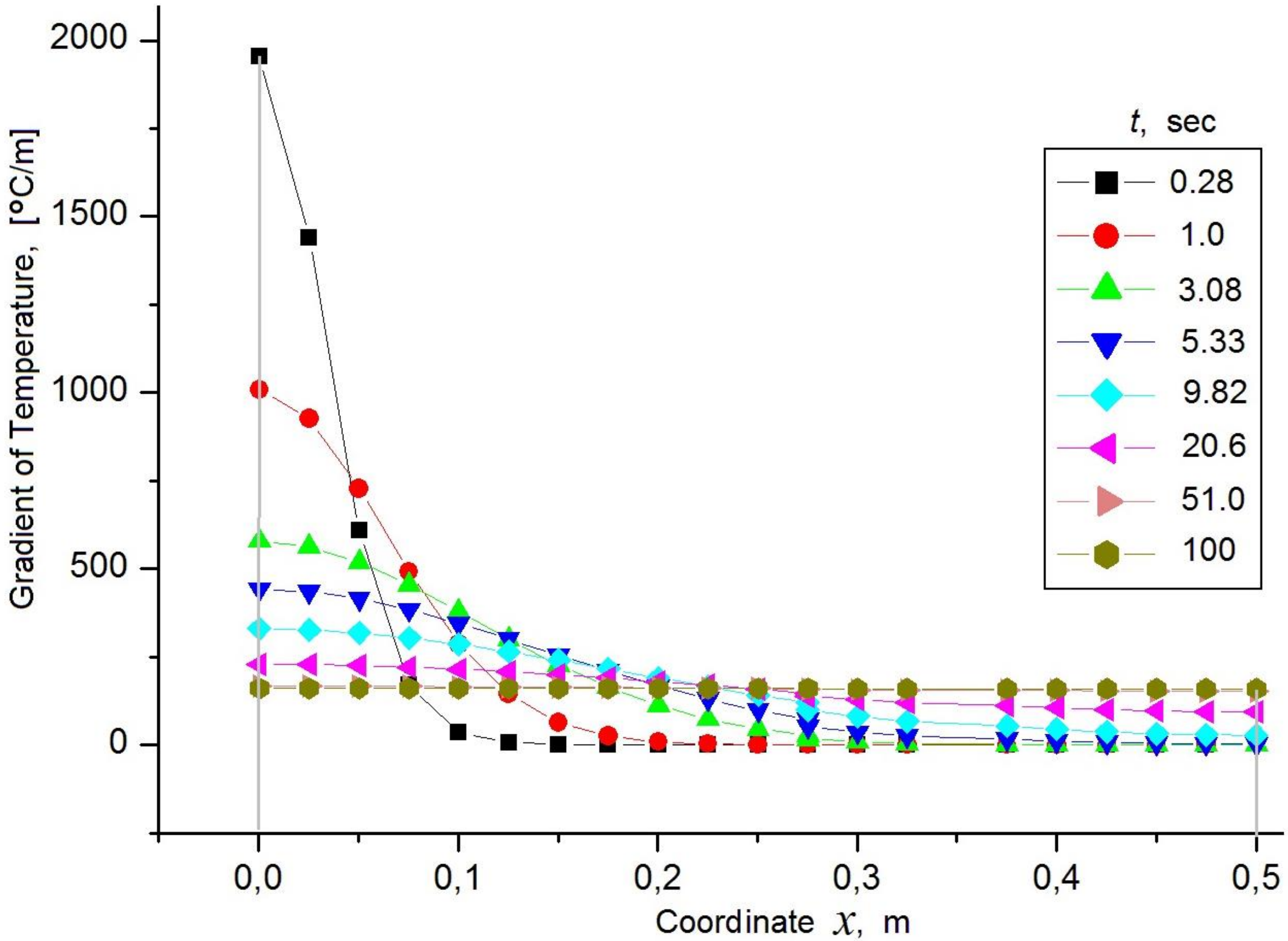
Scale of dimension *X* is reduced in 10 times, t_{MAX} is reduced in 100 times.



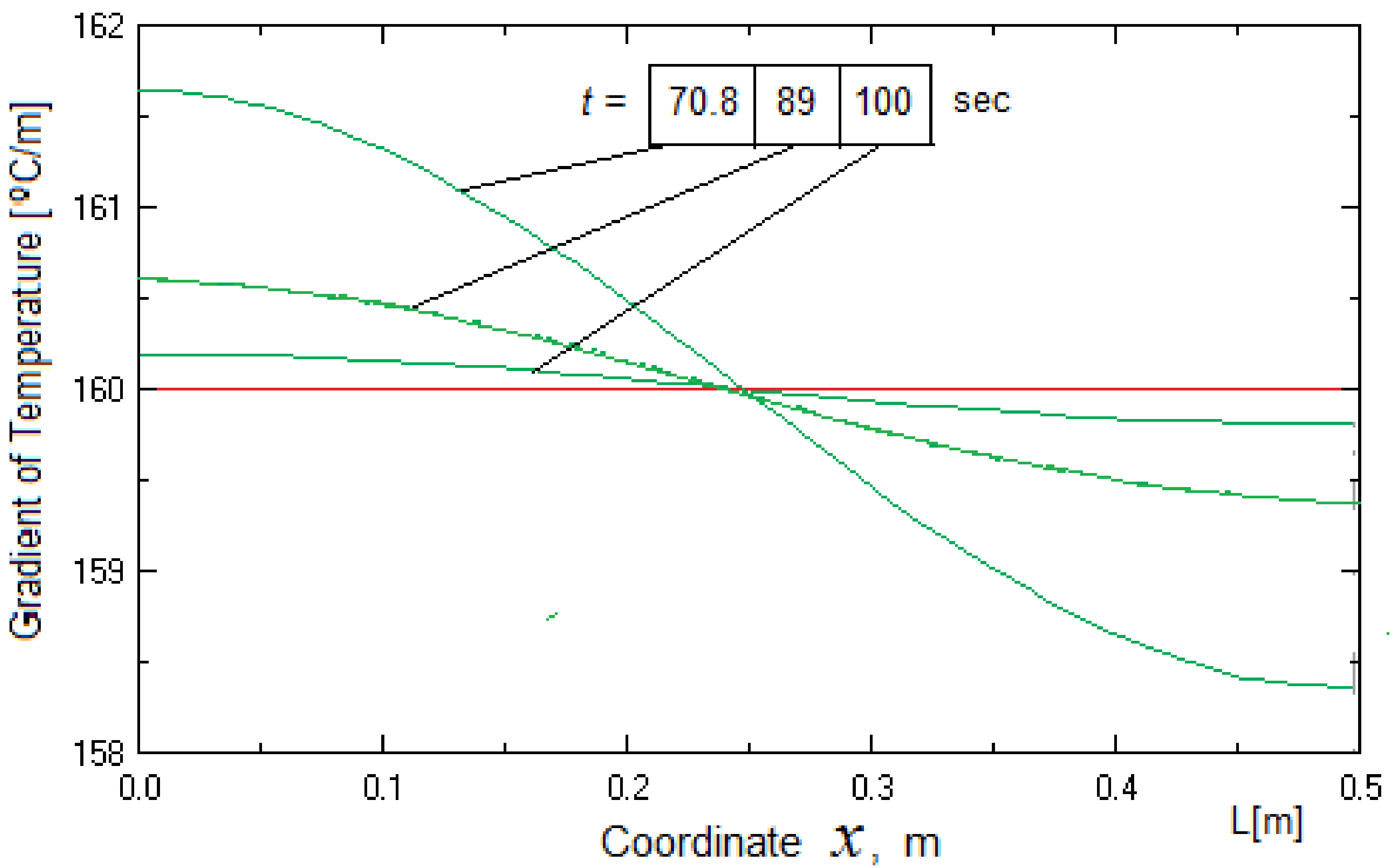
Gradient of Temperature for scaled model (L = 0.5 m).

D = 0.002 m^2/sec. T_1 = 80°C, T_2 = 0°C, t_{MAX} = 100 sec.

It is seen that with reduced length of sample to 0.5 m
the heat flux automatically is increased in 10 times, what is general condition
for the similarity of solution on the temperature.



Gradient of Temperature at final instants of time (L = 0.5 m).
D = 0.002 m2/sec, *T*1 = 80°C, *T*2 = 0°C, *tMAX* = 10000 sec.
Final deviation of ***grad T*** from uniform value is less than 0.1%.



CONCLUSION 1. For the achievement of solution full similarity in the simulation of scaled model of transient heat transfer it is necessary:

1) to perform the equality of general criterion similarity

$$D_1 \frac{t_{1bas}}{X_{1bas}^2} = D_2 \frac{t_{2bas}}{X_{2bas}^2}$$

(here t_{1bas} and t_{2bas} are the basic time for both processes, or time for access of stationary state);

2) to provide the proportional change the heat flux in direction of the heat diffusion with a change of dimension:

$$q_1 = \frac{\Delta T_1}{L_1}; \qquad q_2 = \frac{\Delta T_2}{L_2};$$

from the condition $\Delta T_1 = \Delta T_2$

it follows a correlation for heat fluxes: $\frac{q_1}{q_2} = \frac{L_2}{L_1}$.

This correlation is able to serve a condition for identity of temperature (*T*) distribution in absolute meanings.

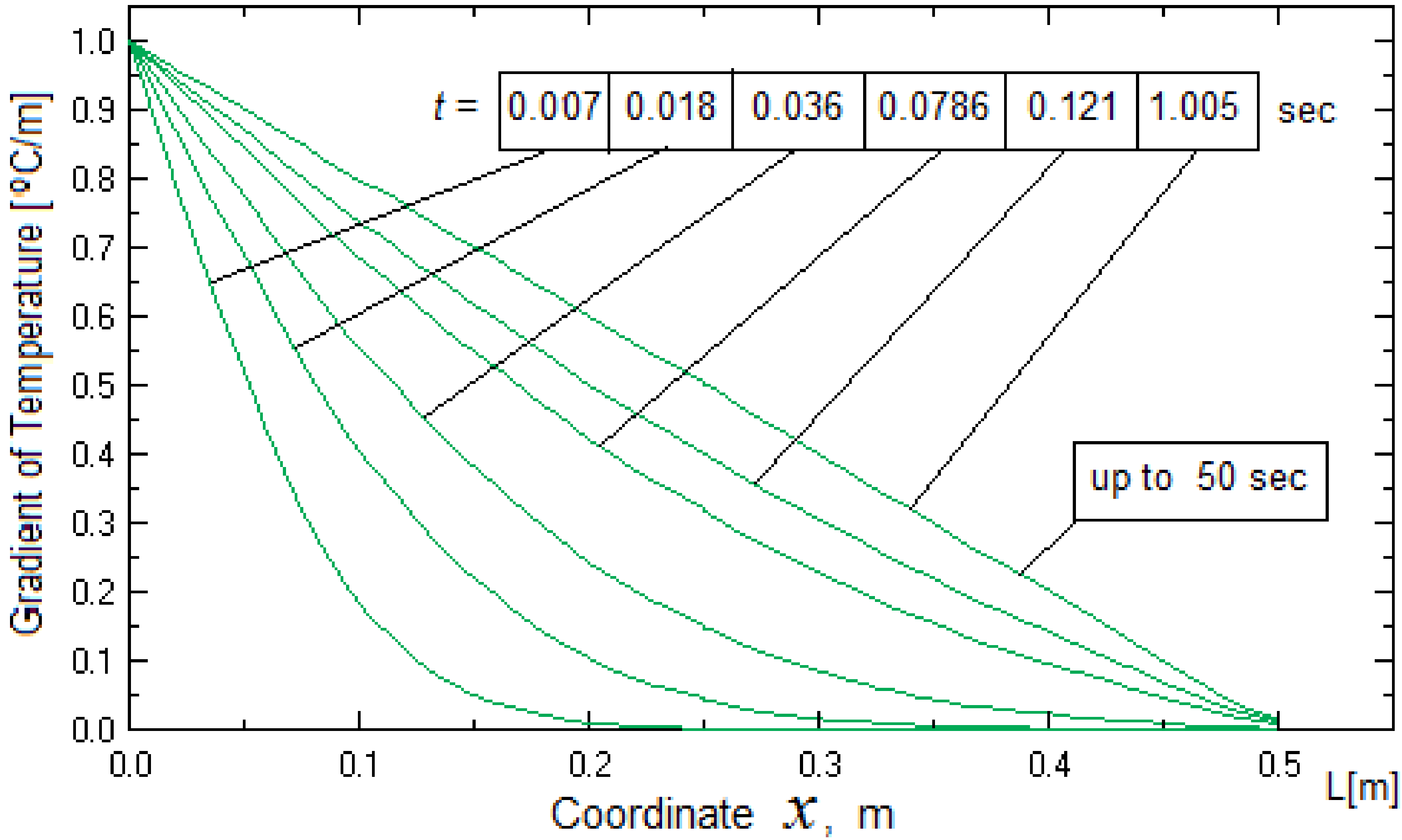
It will be shown in the next example.

Formation of Gradient of Temperature

at 1st second when its value is given as constant at the left edge of body.

L = 0.5 m, D = 0.417 (m^2/s), q_0 = 10000 (W/m), T_2 is free, t_{MAX} = 50 sec.

T_1 depends on the heat flux intensity and time.

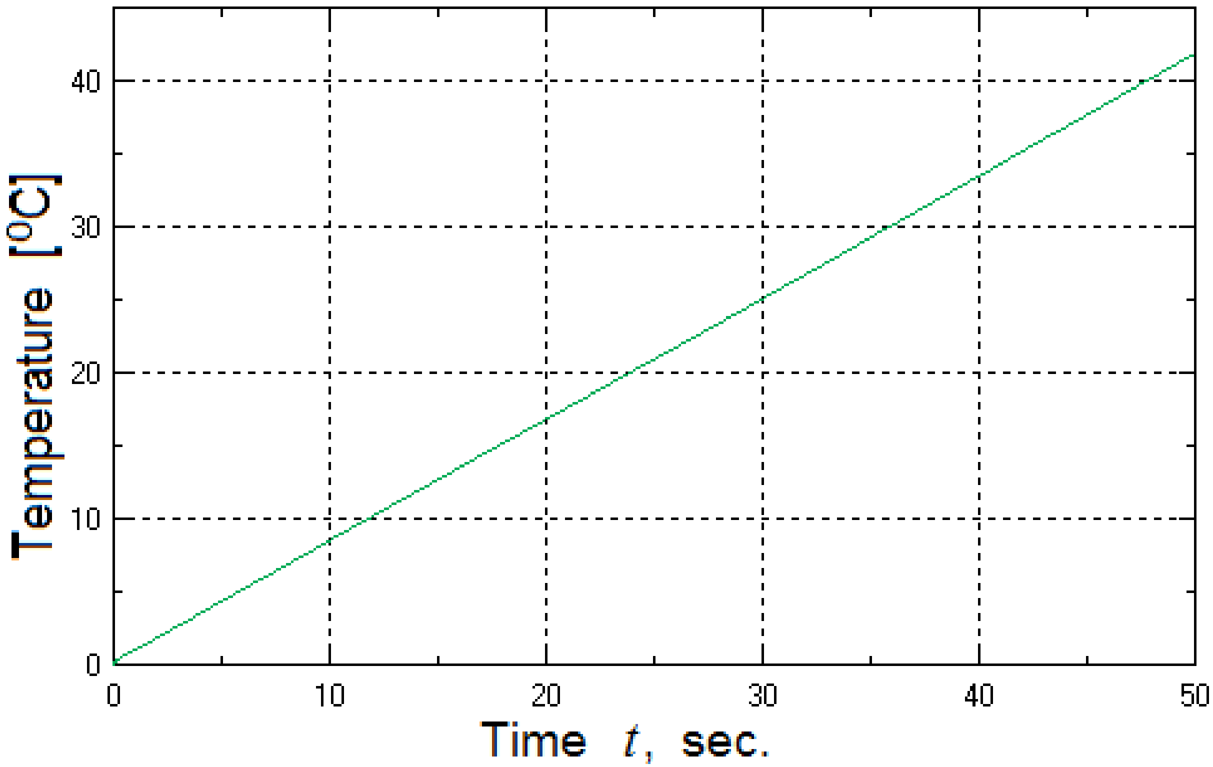


Temperature at the left edge of body at the constant value of heat flow.

L = 0.5 m, D = 0.417 (m^2/s), q_0 = 10000 (W/m), t_{MAX} = 50 sec.

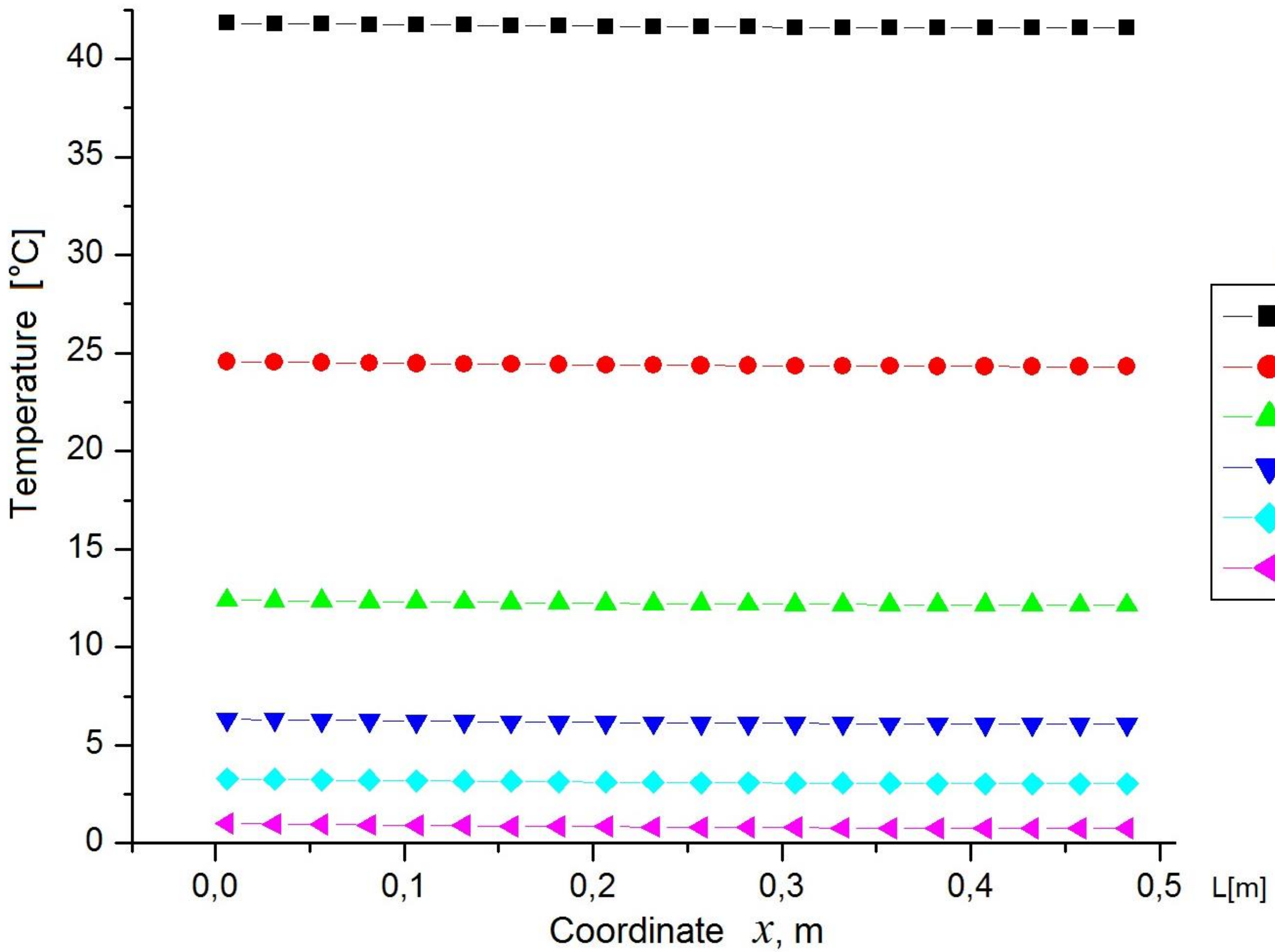
{The similar graph of temperature was obtained also for

L = 5.0 m, D = 0.417 (m^2/s), q_0 =1000 (W/m), t_{MAX} = 5000 sec.}

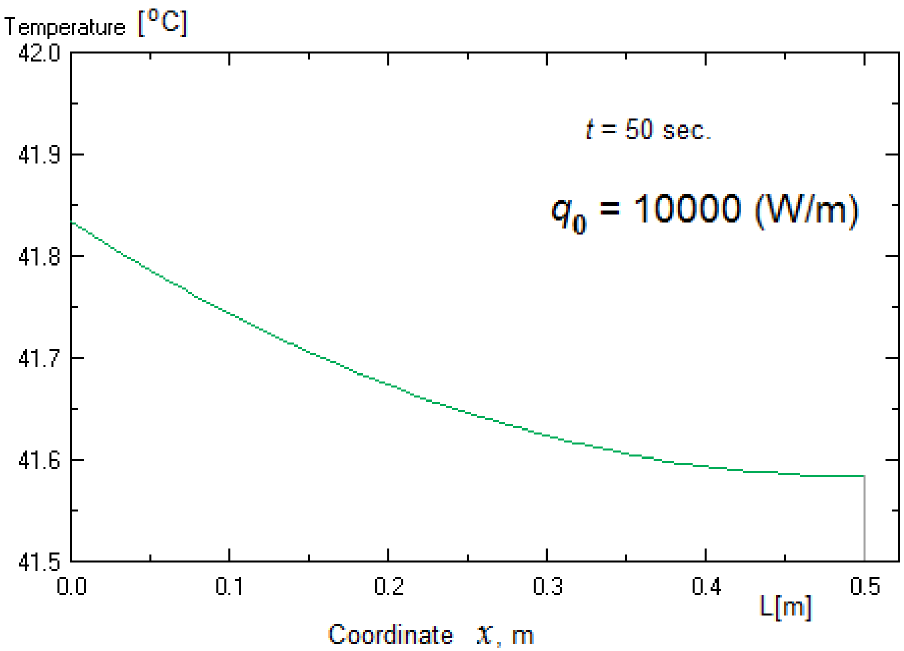


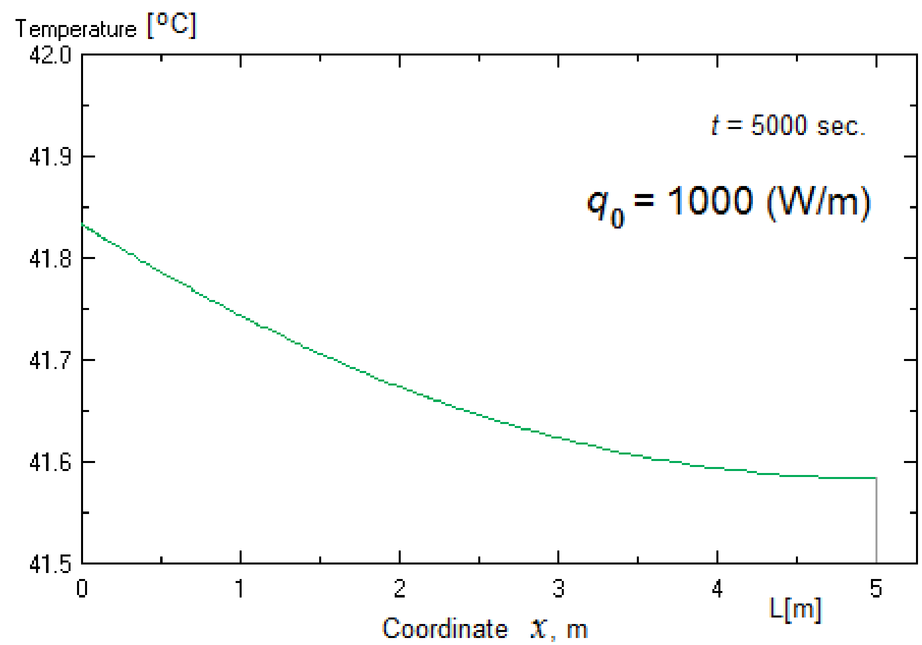
A temperature distribution along the sample of L = 0.5 m during its time growth at the left side of model.

{Similar distribution of temperature was obtained in the scaled model with L = 5 m at changed time of integration (to t_{MAX} = 5000 sec) and changed value of heat flow (to 1000 W/m instead of 10000 W/m).}



Comparison of temperature distribution along the X-axis at the final time instant gives a confirmation of both processes similarity.





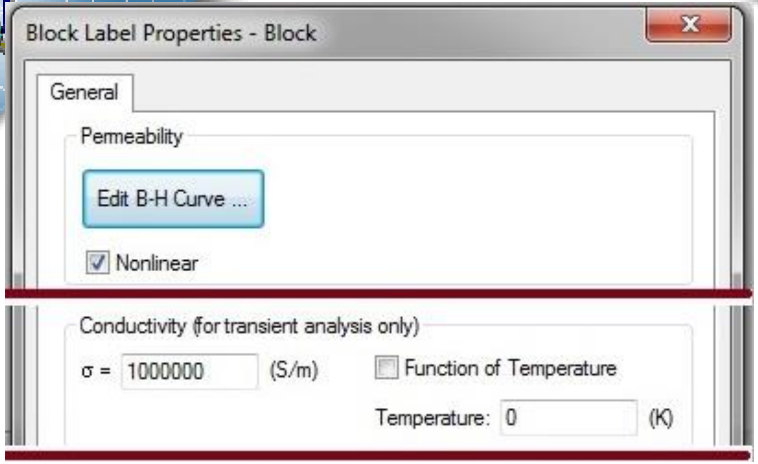
If we are able to reach a **similarity of process** on the base of criterion of similarity with a control of gradient of temperature equality **in the final instant of time for stationary state** of process, absolutely by the same way it is possible to reach the similarity for process at any another duration when the conditions of similarity for space and time parameters of models are fulfilled jointly with boundary conditions.

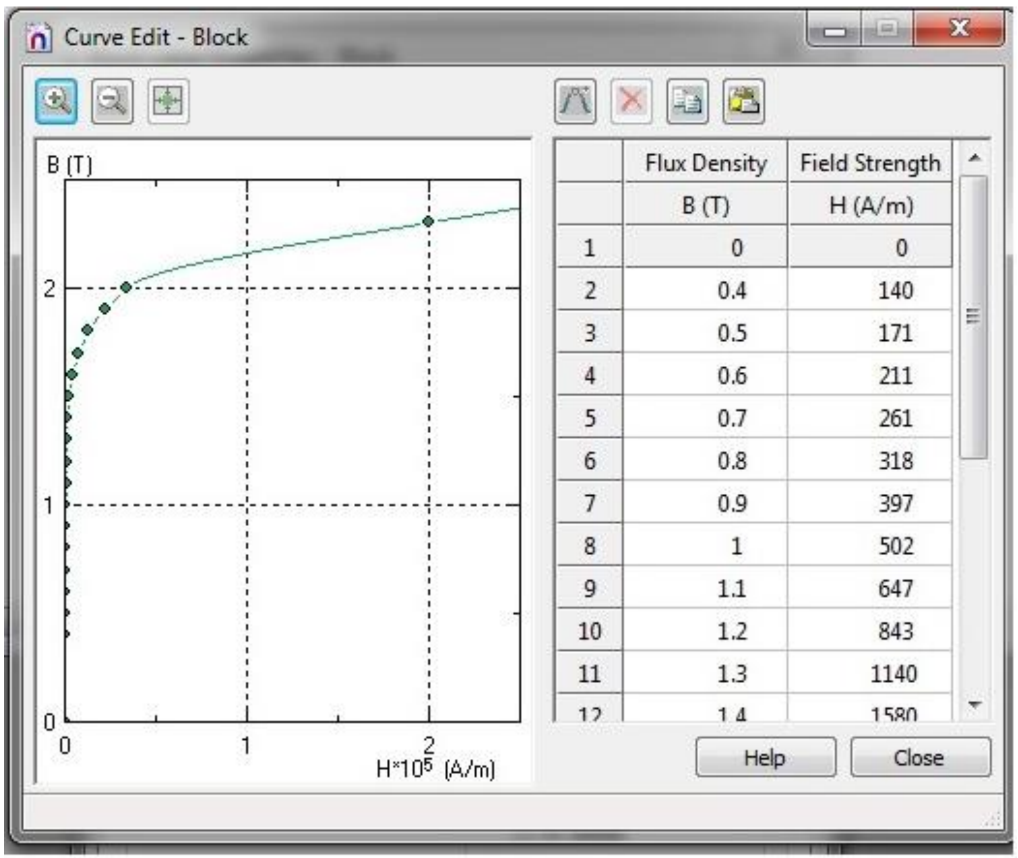
Similarity of solutions for equation of magnetic field diffusion

In the linear medium which has a constant electrical conductivity and constant magnetic permeability the solution of the transient magnetic field diffusion equation has the same character as the equation of the heat diffusion. Some difference occurs in the formulation of boundary conditions.

In the magnetic field problem the y-component of vector magnetic potential A_y plays the same role in boundary conditions as the temperature T in the heat transfer problem.

At the same time, the magnetic induction which for one-dimensional case is equal to $B_z = \partial A_y / \partial x$, plays the same role in the solution as the heat flux q in the heat transfer problem ($q_x = -\lambda \, \partial T / \partial x$).





For the investigation of the similarity of scaled model at the magnetic field diffusion the steel of not very high magnetic properties was taken as material of body in the same geometric models as for the transient heat transfer process.

Samples have the same length 0.5 m or 5.0 m.

When the same magnetization curve of material has input into the calculation model, it can be a guarantee of solution full similarity at transformation the space and time characteristics of diffusion process in non-linear magnetics with energy losses.

Boundary Conditions along the Edges. Sample 1 (L = 5 m).

Edge Label Properties. Problem: **Transient Magnetic Field**.

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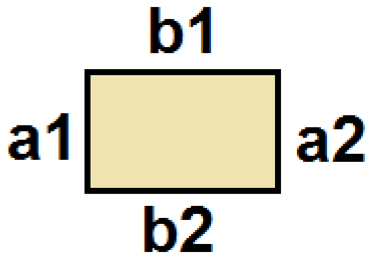
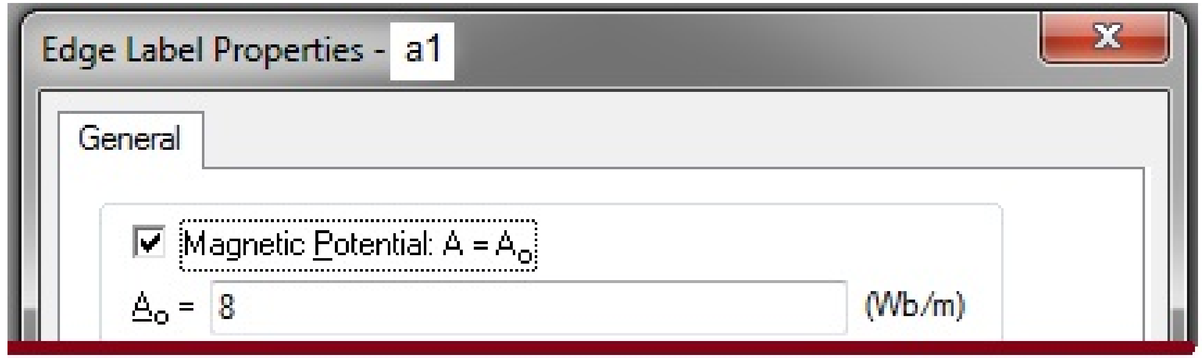
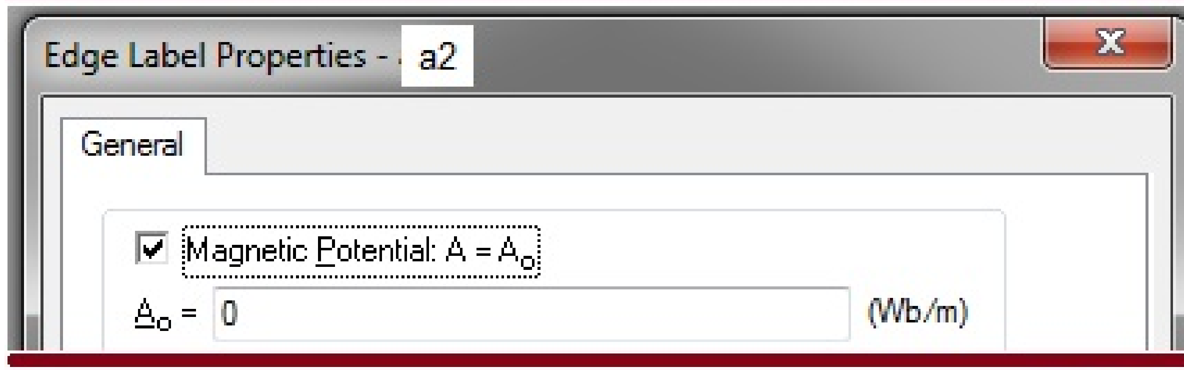


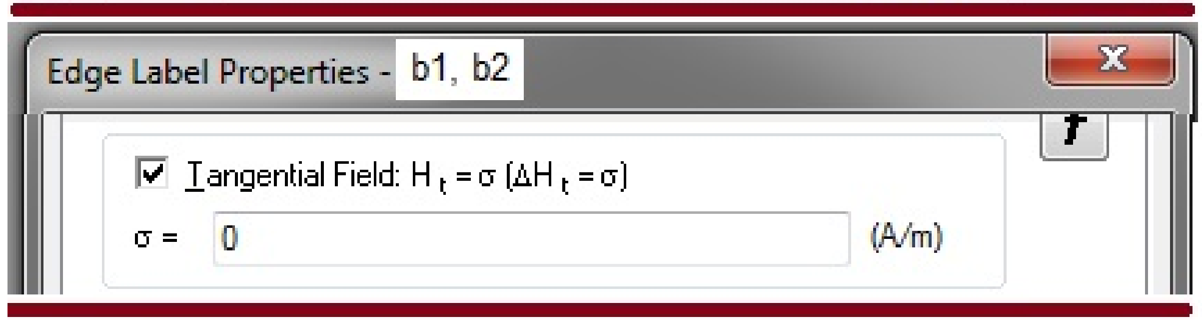
Along Label **a1**



Along Label **a2**

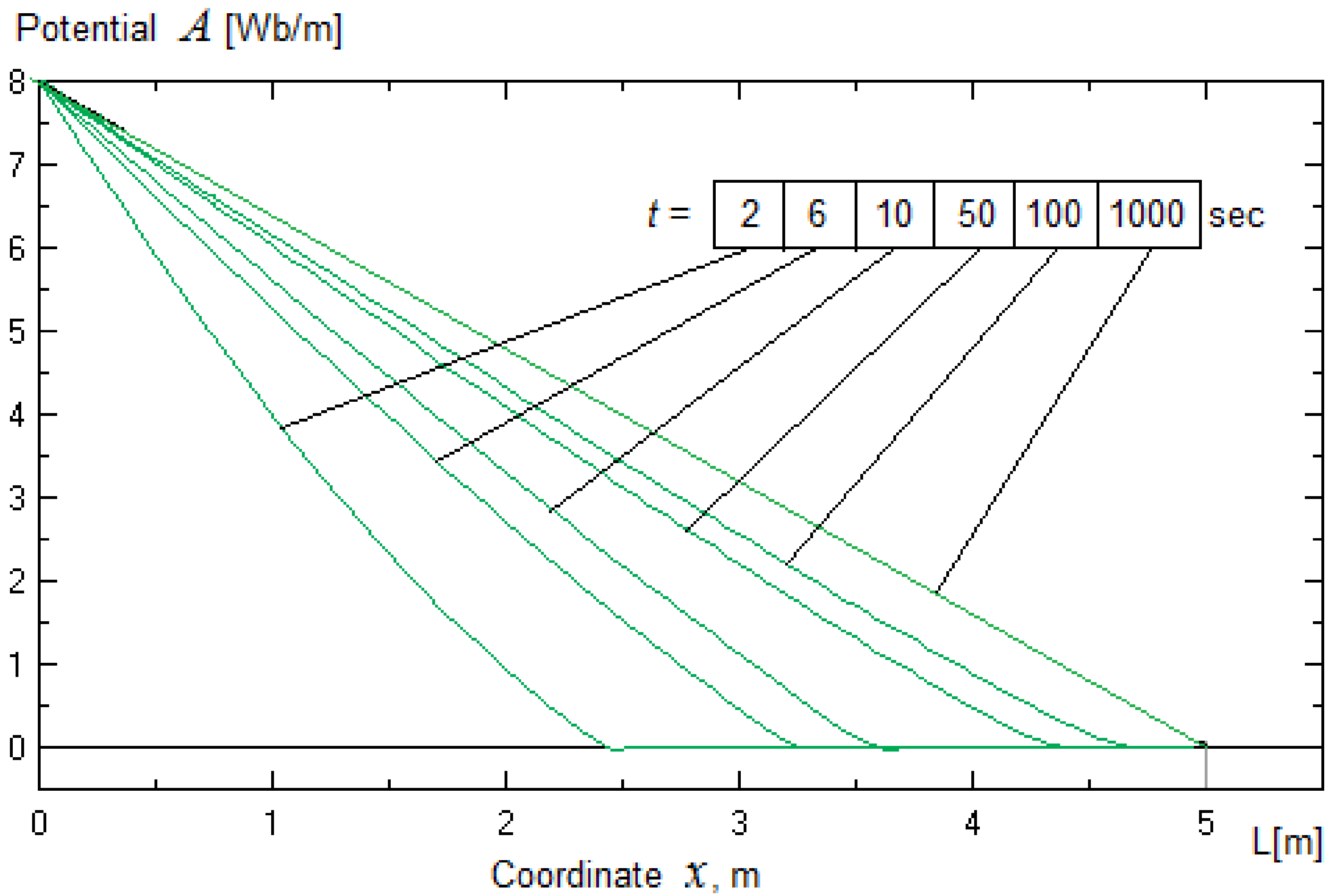


Along Labels **b1, b2**



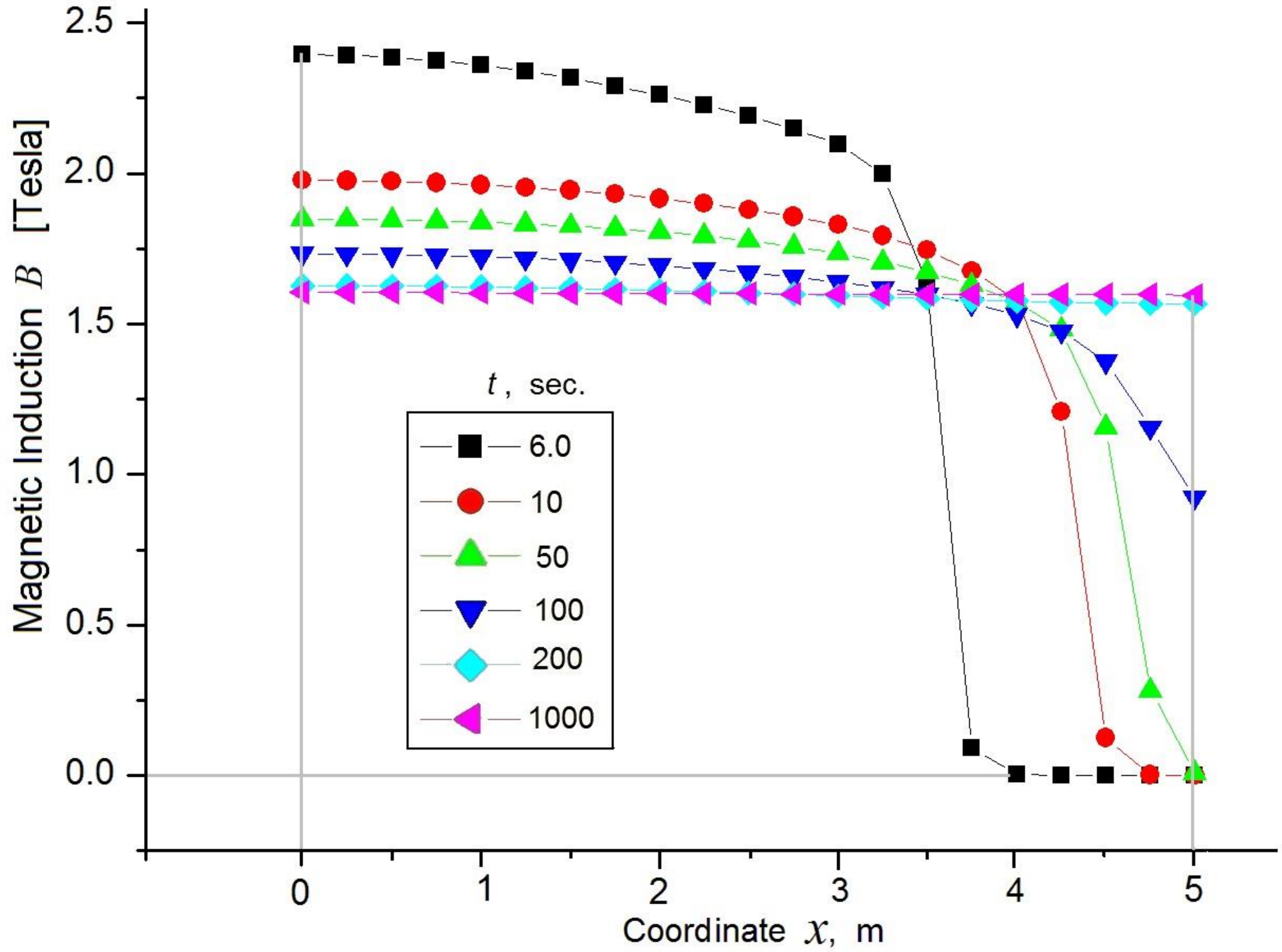
$$A_1 - A_2 = 8.0 \text{ [Wb/m]}$$

The graph shows the formation of magnetic potential distribution in the early moments of process in Sample 1 (L = 5.0 m).
In the late time interval 300…1000 sec a distribution of potential is stable.
The same picture was obtained at the solution of equation for Sample 2 (L = 0.5 m) with change of distance scale to 0…0.5 m and change of time scale to 0…10 sec.

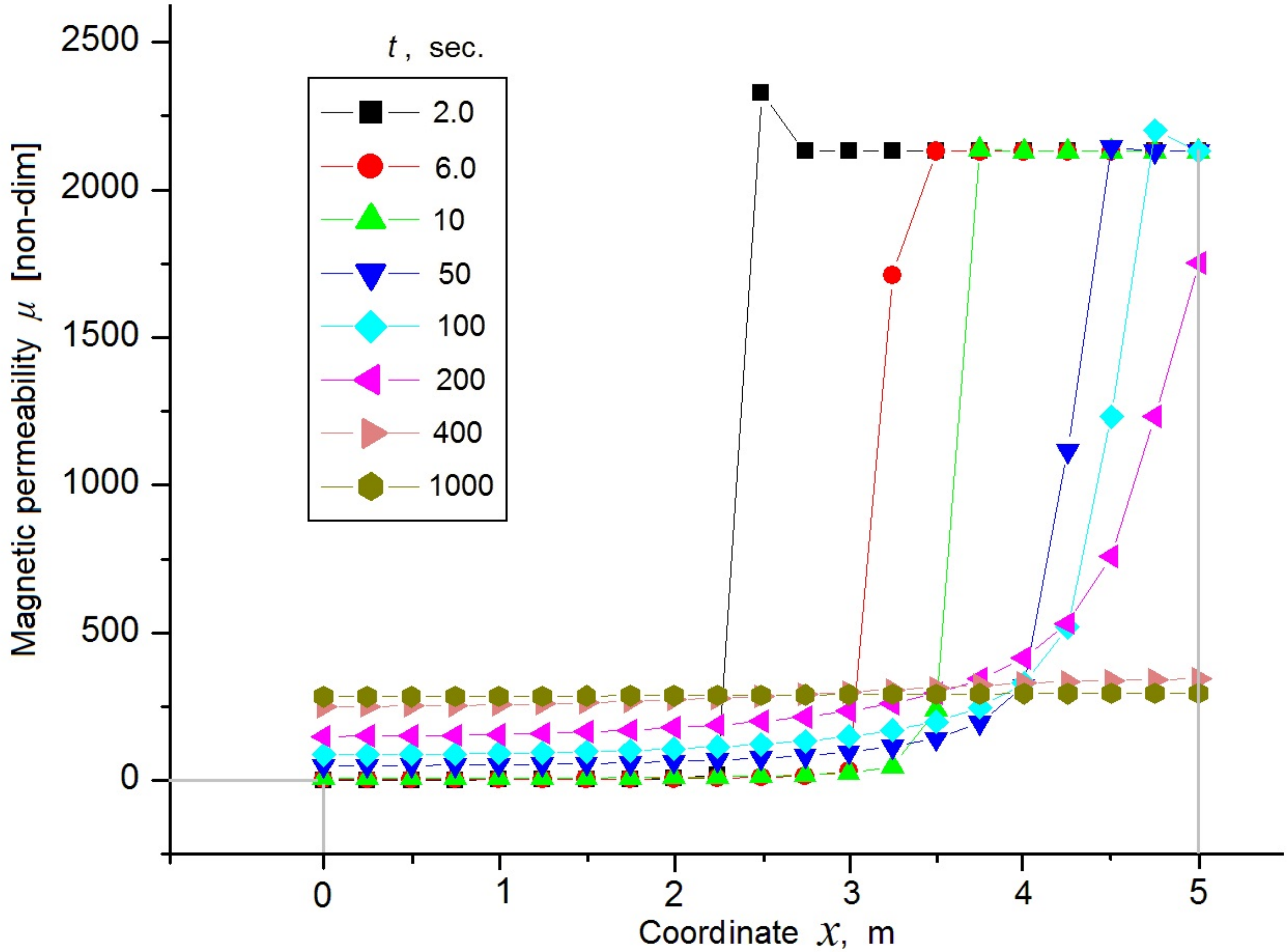


Distribution of magnetic induction along the Sample 1 (L = 5.0 m) has the specific zone of saturation with a moving border of depth.

Process is going to the stationary state at $t \to 1000$ sec.



In the initial stage of process the steel has a zone of deep saturation with moving border of depth while in the rest part of sample steel is not saturated.



Field distribution at the similar transformation of model

At the change of dimension and time of process in accordance with criterion of similarity for accepted example of simulation

$$[t_{1MAX}/t_{2MAX} = (X_1 / X_2)^2 = 100]$$

a field distribution in a scaled model is similar to first one in original due to the equal level of magnetic induction in spite of non-linear magnetic properties of medium.

It is achieved due to boundary conditions which provide the equal meanings of magnetic flux density across the cross section of model:

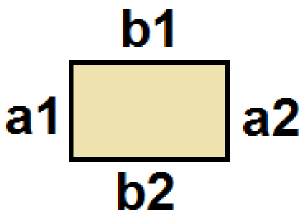
$$B_z = \Phi/S = (A_1 - A_2) / L \quad [Wb/m^2],$$

where L is a dimension of model along a diffusion direction (axis X) .

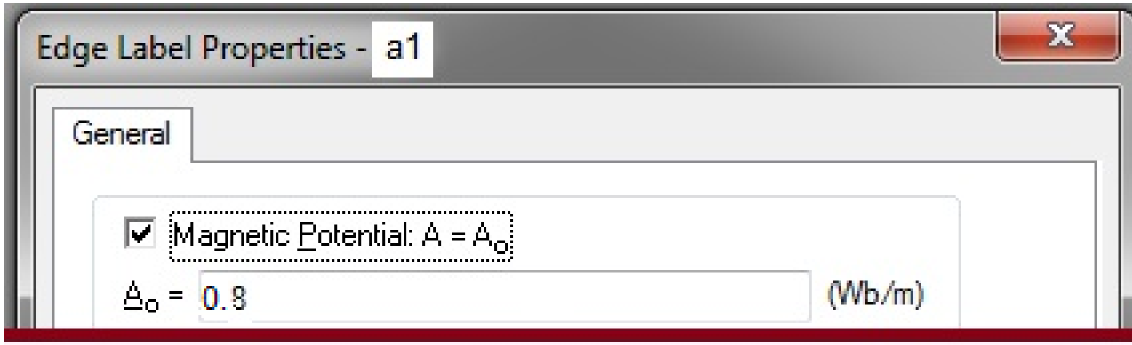
Equality of magnetic fluxes in both models which are waited in the stationary state can be considered as the additional condition of similarity which provides the equal level of field and similar influence of material non-linearity on the field in scaled models.

Sample 2 (L = 0.5 m).
Boundary Conditions along the Edges.

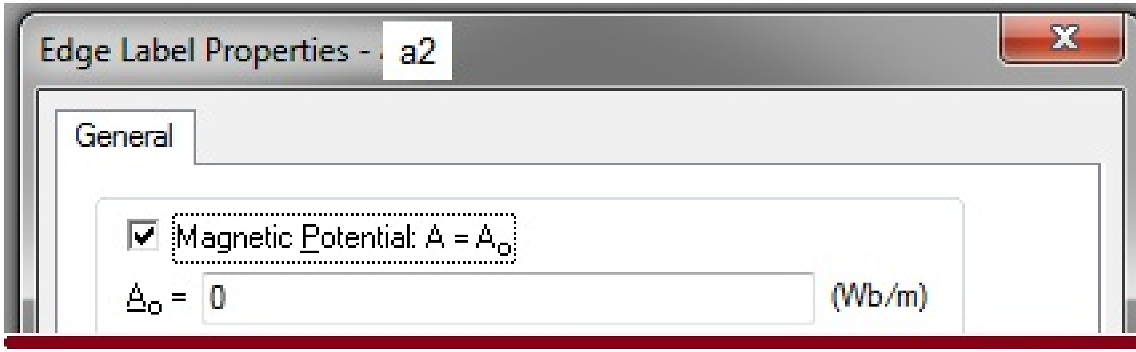
Edge Label Properties. Problem: **Transient Magnetic Field**.



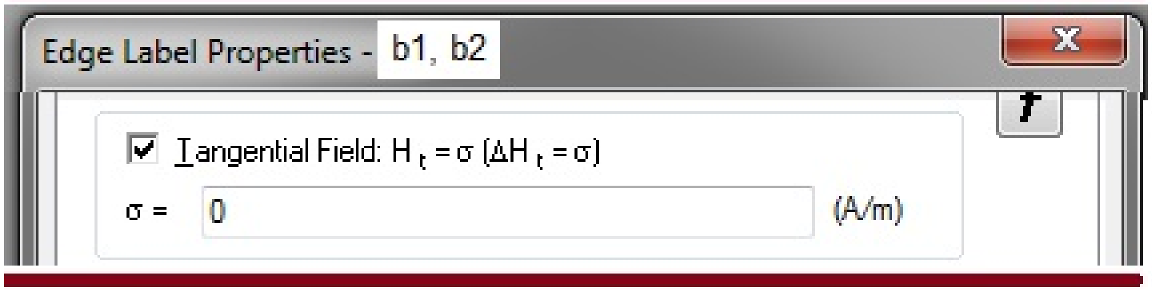
Along Label **a1**



Along Label **a2**

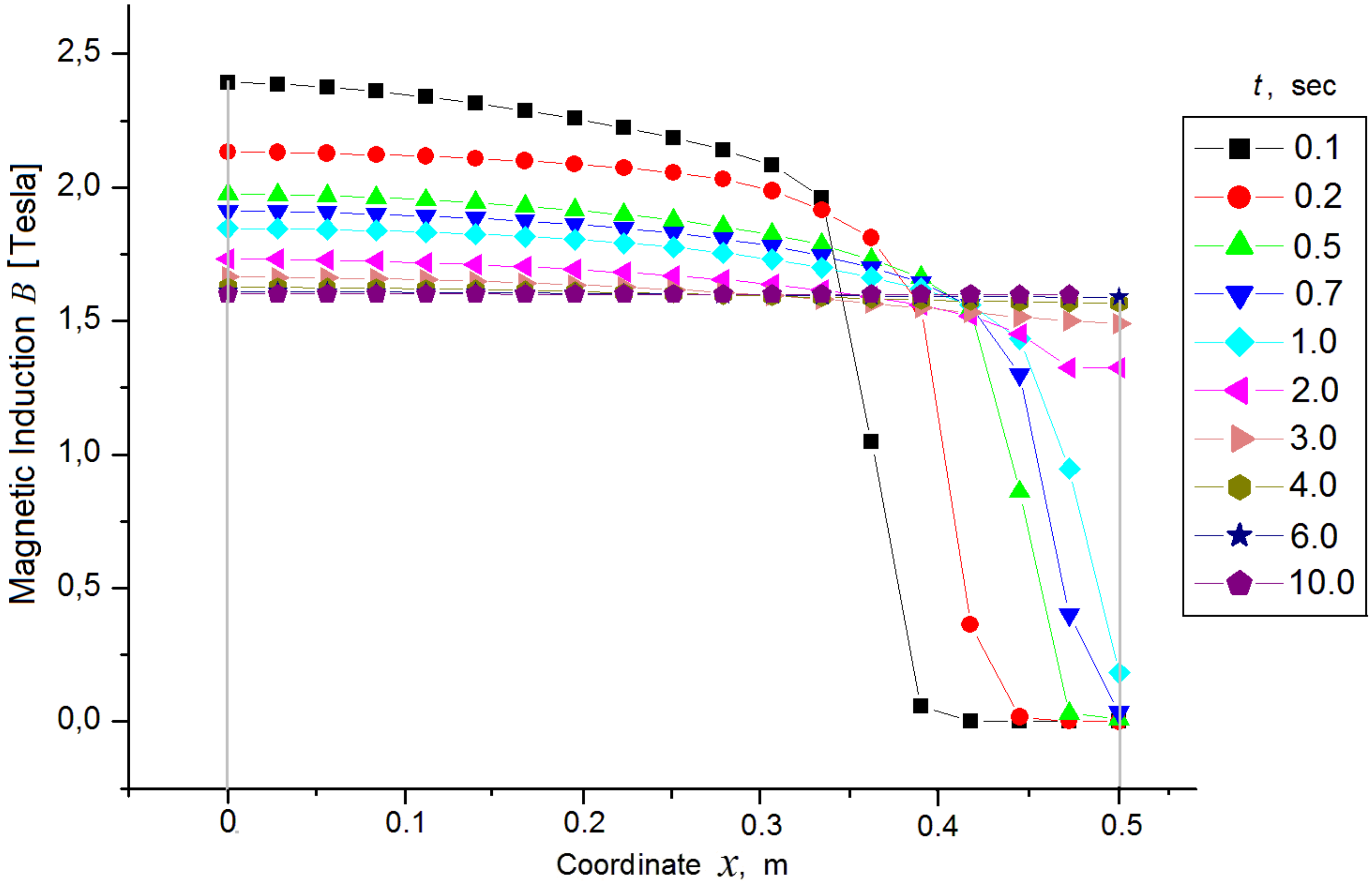


Along Labels **b1, b2**

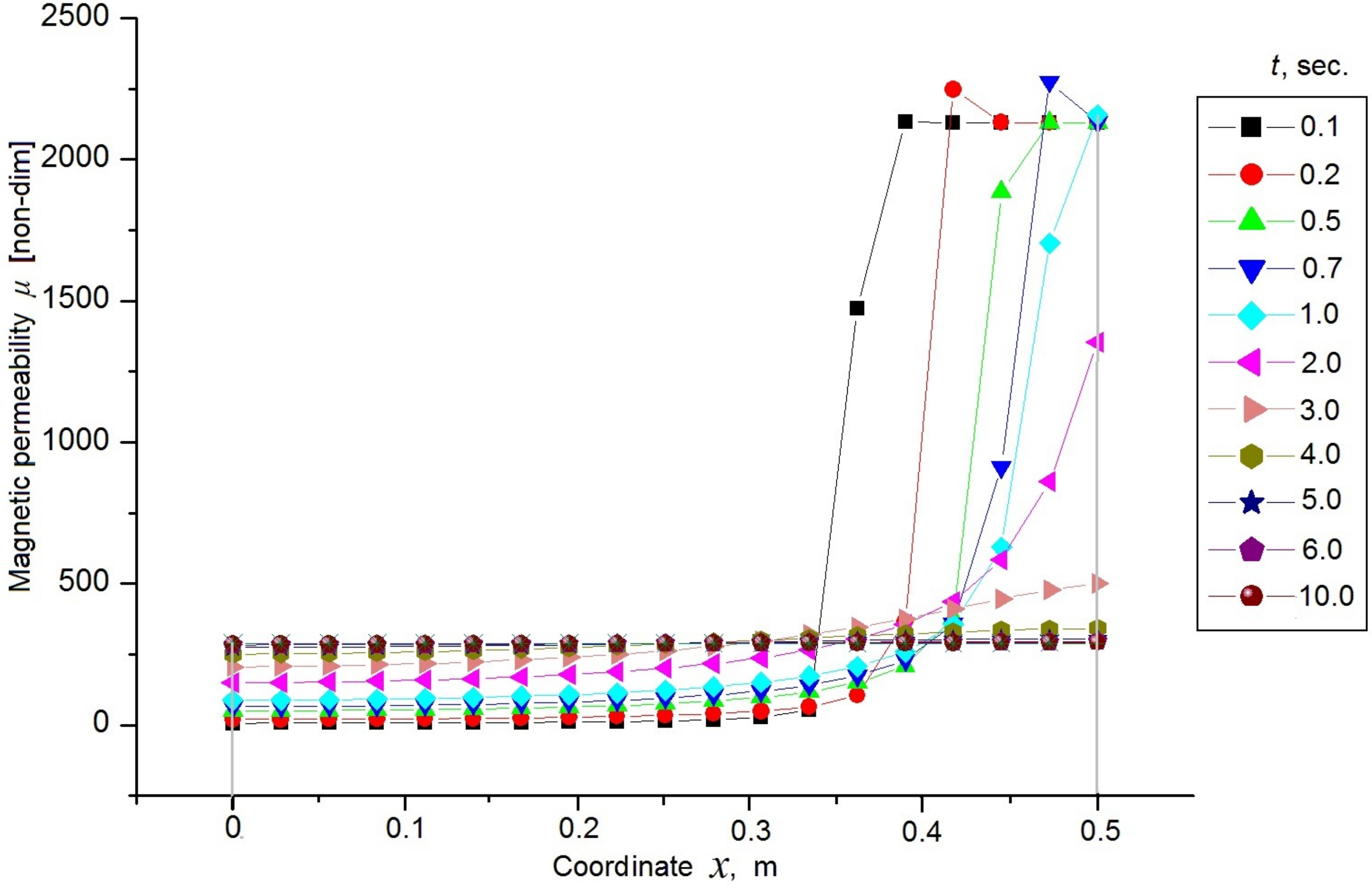


$$A_1 - A_2 = 0.8 \text{ [Wb/m]}$$

Distribution of magnetic induction in the Sample 2 (L = 0.5 m) has not a difference with respect to Sample 1 (L = 5 m) in the changed scale of time and distance.



Distribution of magnetic permeability along the length of Sample 2 (L = 0.5 m) principally is the same as for Sample 1 (L = 5.0 m) in the changed scale of time and distance.



Is the coefficient of magnetic field diffusion comparable for solutions under consideration?

For the comparison of diffusion coefficient for material with non-linear coefficient diffusion the meaning of coefficient in the stationary state (after finish of diffusion) can be used.

In both cases (Sample 1, Sample 2) the stationary meaning of magnetic permeability at same magnetic induction B_z = 1.6 T is equal to $\boldsymbol{\mu}$ = 289.4 $\boldsymbol{\mu_0}$.

Electrical conductivity is $\boldsymbol{\sigma}$ =1e06 Sm/m.

So, for both cases it is possible to believe that coefficient of diffusion is

$$\boldsymbol{D = 1/[\mu \cdot \sigma]} =$$

$$= 1/[289.4 \cdot 1.26e{-}06 \cdot 1e06] = 0.0274\ [m^2/sec].$$

CONCLUSION 2. The simulation of magnetic field diffusion in linear medium has not a principal difference with a transient heat transfer.

Analogies between the values of field:

temperature T $\rightarrow$ magnetic potential A ;

heat flux q $\rightarrow$ density of magnetic flux B.

Simulation of magnetic field diffusion in the non-linear magnetics at presence of electrical conductivity can be similar in two geometrically similar samples at fulfillment of the next conditions of similarity :

1) for the space and time characteristics of samples

   (characteristic dimension X_{bas} , duration of process T_{bas});

2) for coefficient of diffusion D estimated for the state of finished diffusion (this is connected with previous condition);

3) for density of magnetic flux B estimated for the state of finished diffusion (it is need for the correct account of non-linearity of medium).

The general condition of model similarity saves here its form:

$$D_1 \frac{T_{1bas}}{X_{1bas}^2} = D_2 \frac{T_{2bas}}{X_{2bas}^2}$$

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