METHOD OF MODELS SCALING AT SIMULATION IN THE PROGRAM QUICKFIELD ON THE BASE OF THE THEORY OF SIMILARITY

(in application to dynamic processes of the fields diffusion)

At the simulation of the processes of physical fields diffusion (i. e. propagation of the flows of mass, heat or electromagnetic field) in the extremely small or, vice versa, tremendously large of dimensions a duration of these processes in time also can be or extremely small or extremely large. There is not necessary to realize a simulation exclusively in real scale of the object dimensions and time duration of process. Due to application of the theory of similarity, it is possible to make easier the task of simulation via transfer of problem to safe for user space and time frames. The essence of such approach consists in the next.

When we have used the program QuickField for simulation of non-stationary processes of heat transfer or electromagnetic field penetration we build the image of model in the graphical editor, then set the boundary conditions and parameters of medium: for the thermal problems that are coefficient of heat conductance λ, specific heat capacity C and density of medium ρ; for the electromagnetic problems that are magnetic permeability μ and coefficient of electrical conductivity σ. At non-linear properties of medium these parameters can be introduce in the form of some functional dependencies. In the simulation we have used natural units of dimensions and time, what is important.

The theory of simulation enables to make transfer from the real dimensions of object and real duration of process to other dimensions and another duration saving the picture of process progression in the time and space to be similar to one which occurs in the real object. This transition is based on the fact that for the similarity of processes it is not necessary to keep equal all separately taken parameters, only necessary to provide the equality of some combination of these parameters which is called as criterion of similarity. For the homogeneous and isotropic media the main characteristic parameter which defines a progression of non-stationary processes is a coefficient of the field diffusion. For thermal processes it is equal to

$$D_{heat} = \lambda / (C \cdot \rho), \quad [\ m^2 /sec\ ]$$

and for electromagnetic processes

$$D_{mag} = 1 / (\mu \cdot \sigma) \quad [\ m^2 /sec\ ]\ .$$

For the total similarity of processes in the calculation (simulation) model in comparison with primary (basic) model this coefficient must be connected with space and time characteristics of model. To realize it, the so called normalized (i.e. non-dimensional) meanings of coefficient of diffusion for primary model ($D_1^{☼}$) and for the transformed calculation model ($D_2^{☼}$) are used:

$$D_1^{☼} = D_1 T_1 / X_1^2 \; ; \qquad D_2^{☼} = D_2 T_2 / X_2^2 \; .$$

Here indexes 1 or 2 have been associated with primary model and simulation model, respectively; D_1 and D_2 are the dimensional values of diffusion coefficients in one and another models; T_1 and T_2 are the characteristic time duration for one and another models (namely this time must be introduced into the properties of problem); X_1 and X_2 are the characteristic dimension of one and another models. At the simulation of one-dimensional processes of field propagation that is usually the model dimension in a direction of field propagation.

The equality of normalized meanings of coefficients of diffusion is the necessary condition for the similarity of the field picture in two models of different scale:

$$D_1^{☼} = D_2^{☼} \; .$$

By other words, the normalized meanings of diffusion coefficients (and their equality) present the criteria of processes similarity.

In a case when the medium of object of simulation has the non-linear properties, i.e. coefficient of diffusion does not depend on the time, space coordinates and the values of field, this condition jointly with a condition of identity of boundary conditions in both models under consideration serves as quite enough condition for the similarity of the field pictures.

At the simulation of field diffusion processes in the medium with a non-linear properties that id necessary to provide additionally the similarity of functional dependencies of diffusion coefficients on the field values or on the time and space to reach a similarity of processes in the models of different scale. Most simple this additional condition is looking at simulation of the field penetration into ferromagnetic which has the electrical conductivity. To fulfill this condition it is enough to introduce the same curve of magnetization B=f(H) both into the basic model and into the model of changed space and time characteristics. In result, the full identity of numerical values of field can be achieved in both models in the transformed scales of space and time provided the new additional condition about equality of meanings of magnetic induction in the stationary state has been used.

To prove the similarity of the field distribution pictures it is enough to make a prolongation of solution of problem about the field diffusion up to stationary state when a diffusion has stopped. In this state in the thermal problem with boundary conditions of 1st kind a gradient of temperature becomes a constant value along the coordinate while in the problem about non-stationary magnetic field diffusion a distribution of magnetic induction along the coordinate becomes uniform. As a similarity of the field has been achieved in the models under comparison at full completion of diffusion process so such similarity is possible also at earlier time steps with corresponded change of scale for time and coordinates.

Examples.

1. In the file Heat_Diff_1.pbm the problem of non-stationary heating of band length of L_1=5m which made of uniform material has been formulated in quasi-one-dimension consideration with a set of coefficient of heat diffusion D_1=5e-04 m^2/sec and boundary conditions of 1st kind: at the initial edge of band (x=0) the fixed temperature t_1=80ºC, and at the far edge (x=L_1) a temperature t_2=0ºC is given. The process duration ("timing") is accepted as T_1=5000 sec. After the end of this time interval some distribution of temperature along the band length is achieved what is shown in the Fig.1.
2. In the file Heat_Diff_2.pbm at the analogous conditions the problem of non-stationary heating of band of the same material at its length L_2=0,5m has been formulated with a changed time duration of process to T_2=50sec but not changed value of the heat diffusion coefficient D_2=5e-04m^2/sec. At the end of timing a distribution of temperature is achieved as shown in the Fig.2. This distribution is completely similar to one shown in the Fig.1.

   It was achieved due to correlation between the characteristic dimension and time of problems was taken in correspondence with normalized meanings of diffusion coefficients, what means $T_1/ T_2=(L_1/L_2)^2$ at $D_1=D_2$.
3. In the file Sample1.pbm in the quasi-one-dimensional consideration a problem of non-stationary diffusion of magnetic field into a band of ferromagnetic material has been formulated using the magnetization curve which is given as Table 1. Electrical conductivity of material is supposed the constant value: σ=1e06Sm/m. The length of band is L_1=0,5m, time duration of process is equal to T_1=10sec. At the edges of band the next boundary meanings of vector potential have been set: at the initial edge (x=0) potential is A_1=0,8Wb/m, and at the far edge (x=L_1) potential is A_2=0. At the end of timing the uniform distribution of

magnetic induction along the length of band was obtained what is shown in the Fig.3. Such distribution of magnetic induction says about finish of the field diffusion into this sample.

4. In the file Sample2.pbm the problem about non-stationary diffusion of magnetic field into the band length of L_2=5m made of the same ferromagnetic material as in the previous problem with a same magnetization curve B=f(H). A difference of boundary meanings of vector potential at the edges of band ($A_1 - A_2$) has been increased up to 8Wb/m (namely, A_1=8Wb/m and A_2=0). Due to this increase the same value of magnetic induction in the stationary state (1.6T) is provided likely to previous problem. The similar level of magnetic induction serves a guarantee of similar influence of magnetic properties non-linearity on the field distribution. The obtained distribution of magnetic induction along the length of band is shown in the Fig.4 for different time instants. It is seen the real similarity of the field distribution to the Fig.3. This similarity is more and more close with time going to the stationary state. Some difference in the field distribution is possible at the initial steps of the field diffusion when the formation of saturated on material near the initial edge occurs along with jumped switch of the field at the boundary. The equality of diffusion coefficients for the comparable solutions Fig.3 and Fig.4 can be checked using the parameters μ, σ for stationary state.

CONTENTS OF DIRECTORY "The files of problems"

Problem Heat_Diff_1:	**Nonlinear Sample_1:**
Heat_Diff_1.pbm	Sample_1.pbm
Heat_Diff_1.mod	Sample_1.mod
Heat_Diff_1.dht	Sample_1.dht
Heat_Diff_1.res	Sample_1.res
Problem Heat_Diff_2:	**Nonlinear Sample_2:**
Heat_Diff_2.pbm	Sample_2.pbm
Heat_Diff_2.mod	Sample_2.mod
Heat_Diff_2.dht	Sample_2.dht
Heat_Diff_2.res	Sample_2.res

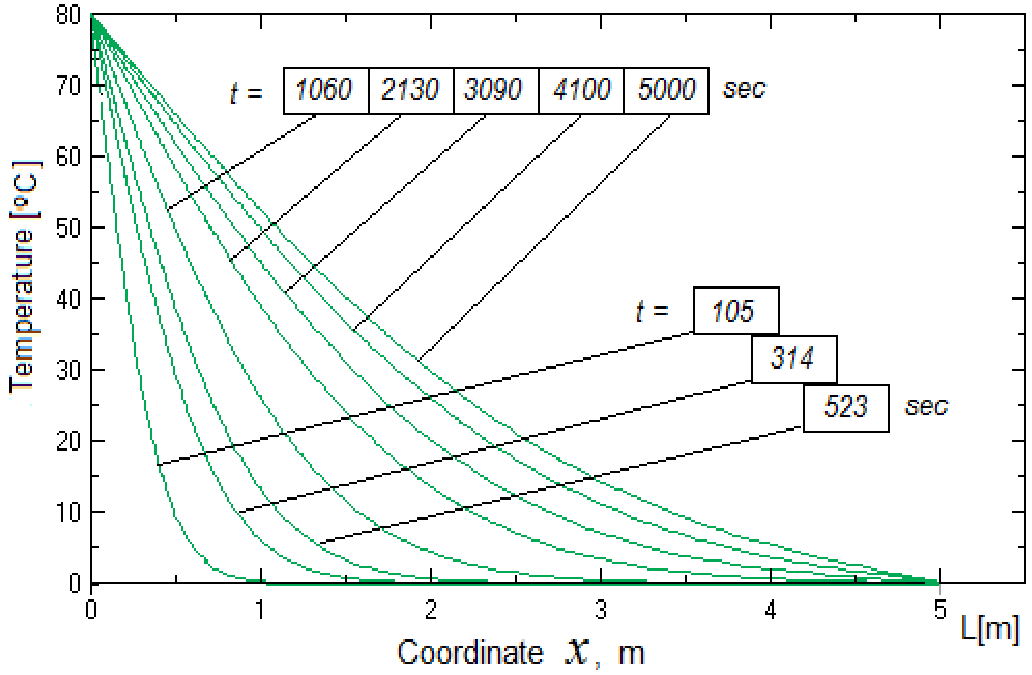

Fig. 1.

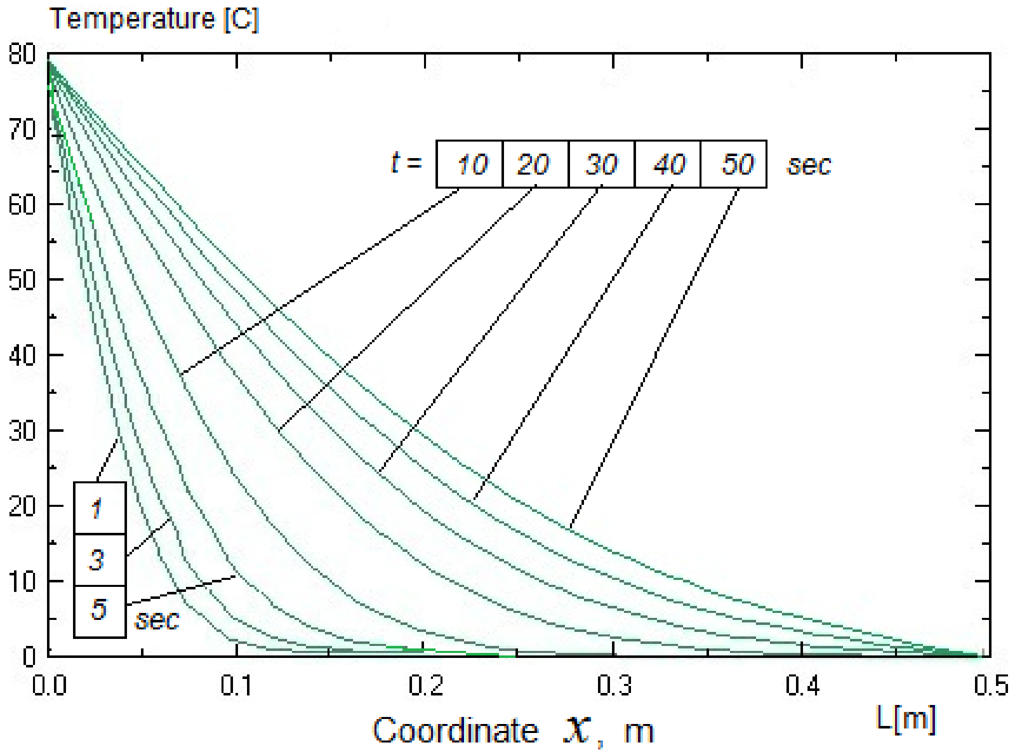

Fig. 2.

Table 1

№№	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
B, Tesla	0	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.3
H, A / m	0	140	171	211	261	318	397	502	647	843	1140	1580	2500	4400	7800	13000	22000	34200	$2 \cdot 10^5$

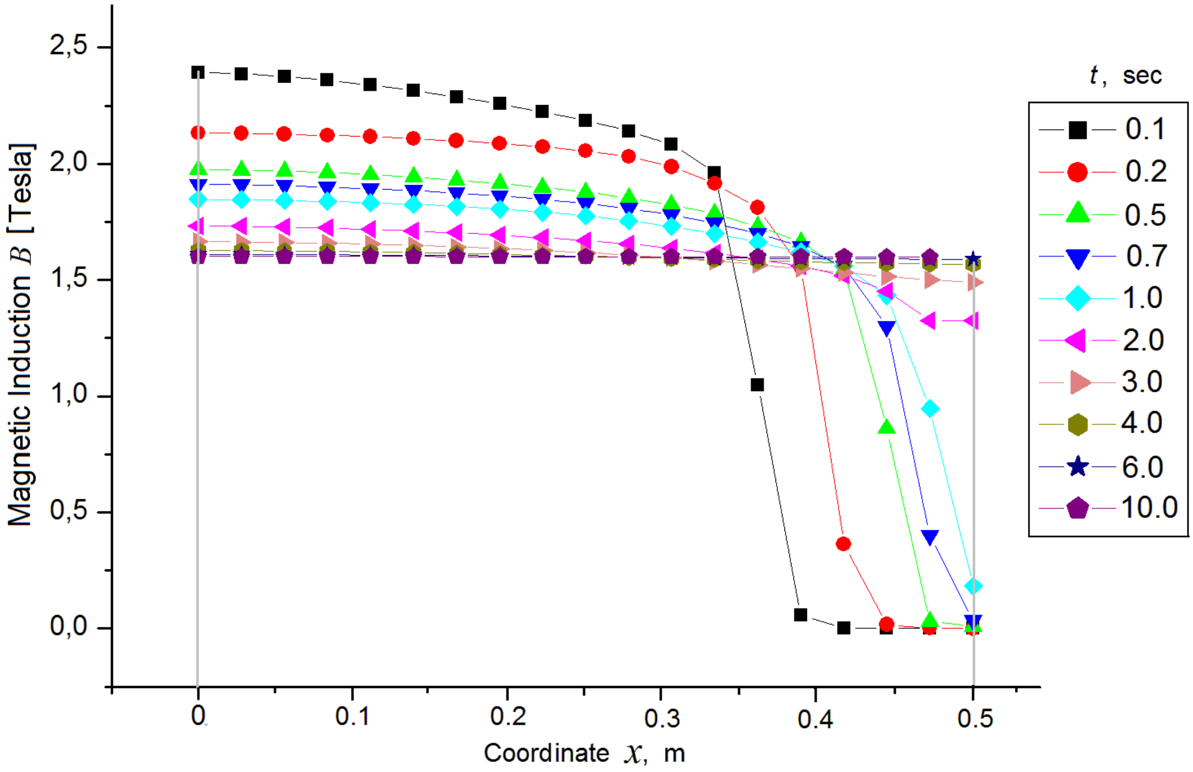
Magnetic Induction B [Tesla]
2,5
2,0
1,5
1,0
0,5
0,0
0.
0.1
0.2
0.3
0.4
0.5
Coordinate x, m
t, sec
0.1
0.2
0.5
0.7
1.0
2.0
3.0
4.0
6.0
10.0

Fig. 3.

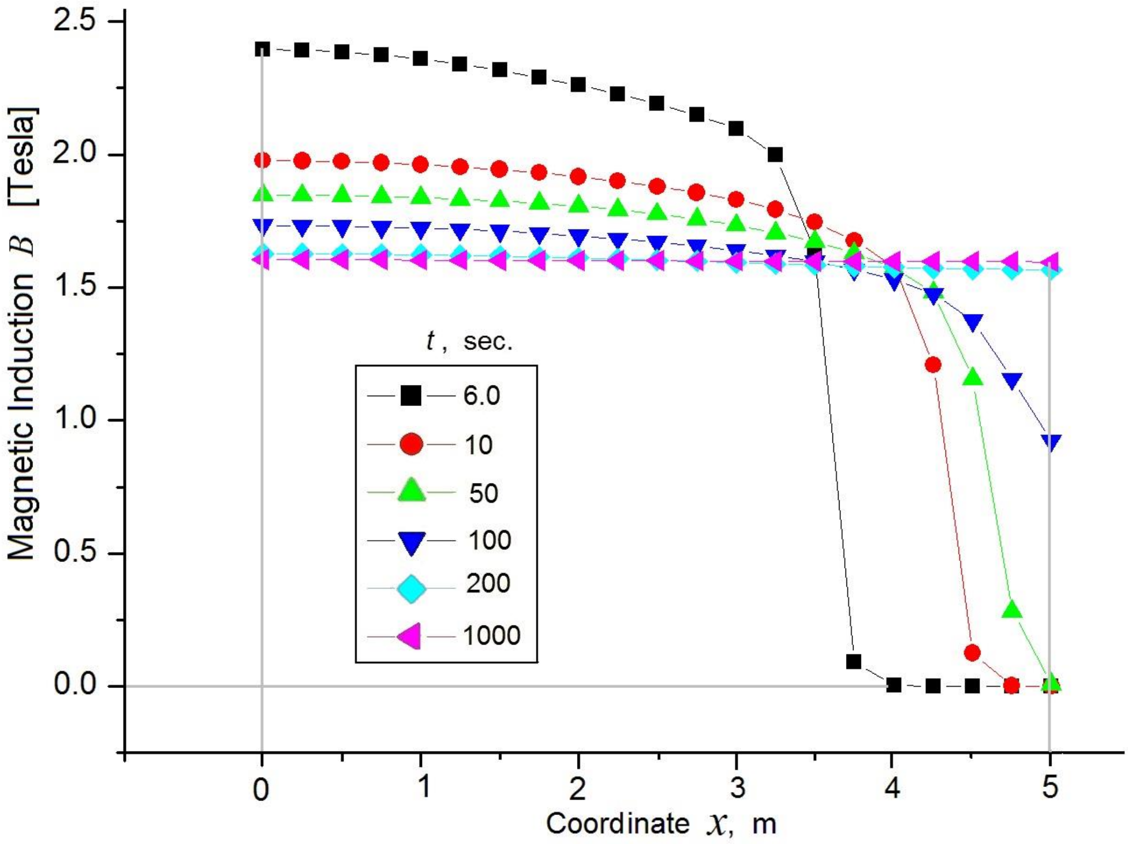
Magnetic Induction B [Tesla]
2.5
2.0
1.5
1.0
0.5
0.0
0
1
2
3
4
5
Coordinate x, m
t, sec.
6.0
10
50
100
200
1000

Fig. 4.