Week 1: Introduction

How to solve the EM field equation using numerical simulation (practical aspects).

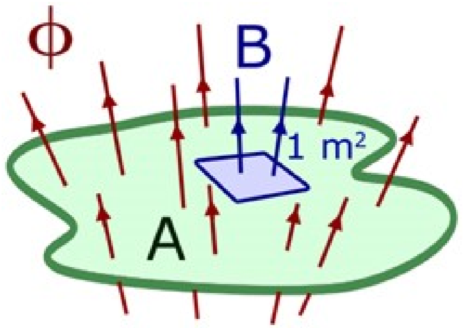
1. Can we numerically solve the full system of Maxwell equations directly?
2. What is problem formulation? Why the well known Maxwell equation is not enough?
3. What we need to add to Maxwell equation for solving?
4. Our current software tool is QuickField. Why?
5. How to jump from Maxwell equations to the first practical problem?
6. What is space discretization, and why it is important?
7. Let’s solve our first problem

What math will we use

Basic concepts of vector calculus

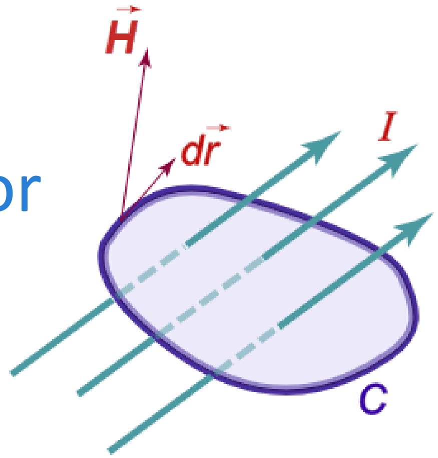
Flux of a vector

$$\Phi = \int_S \vec{\mathbf{B}} \cdot \mathbf{d}\vec{\mathbf{s}}$$



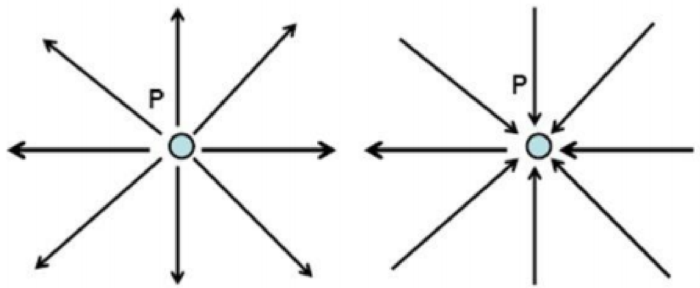
Circulation of a vector

$$MMF = \int_l \vec{\mathbf{H}} \cdot \mathbf{d}\vec{\mathbf{l}}$$



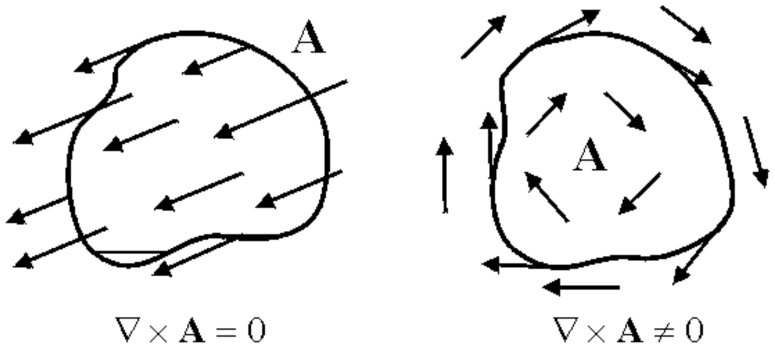
Divergence

$$\mathrm{div}\vec{\mathbf{B}} = \lim_{s\to 0}\frac{\Phi}{V}$$



Rotor (Curl)

$$\mathrm{rot}\vec{\mathbf{H}}|_{\vec{\mathbf{n}}} = \lim_{s\to 0}\frac{MMF}{S}$$



Maxwell Equations (SI convention)

	Integral Equations Operates with finite volumes, surfaces, contours	Differential Equations operates with local quantities: infinite small surface and volume
Gauss's Law	The electric flux leaving a volume is equal to the electric charge inside $\oint_S \mathbf{D}\cdot\mathbf{ds} = q$	The divergence of D-field is equal to the electric charge density, i.e. the charge is a source of electric field $\mathrm{div}\mathbf{D} = \rho$
Gauss's Law for magnetism	The magnetic flux through a closed surface is always zero (no magnetic monopoles) $\oint_S \mathbf{B}\cdot\mathbf{ds} = 0$	The divergence of magnetic B-field is equal to zero, i.e. magnetic field has no sources (= is solenoidal) $\mathrm{div}\mathbf{B} = 0$
Faraday's Law of induction	The EMF induced in a closed contour is equal to the negative of the time rate of change of the magnetic flux it encloses $\oint_L \mathbf{E}\cdot\mathbf{dl} = -\frac{d}{dt}\left(\int_S \mathbf{B}\cdot\mathbf{ds}\right)$	The curl of electric field **E** is equal to the negative rate of change of the magnetic field **B** $\mathrm{rot}\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$
Amper's Law	The magnetic field circulation around a closed loop is equal to the electric current (plus the rate of change of electric field) $\oint_L \mathbf{H}\cdot\mathbf{dl} = I + \frac{d}{dt}\left(\int_S \mathbf{D}\cdot\mathbf{ds}\right)$	The curl of magnetic field **H** is equal to the current density (including the density of displacement current, which is proportional to the rate of change of electric field **D**) $\mathrm{rot}\mathbf{H} = \mathbf{J} + \frac{\partial\mathbf{D}}{\partial t}$

It is not a full story. What should be added?

1. Several material models describing the macroscopic field inside the material.
Any material law is always an approximation

	The simplest (isotropic)	More complicate (anisotropic)	Even more advanced (nonlinear)
Dielectric Model (Permittivity)	$\mathbf{D} = \varepsilon \mathbf{E}$	$\mathbf{D} = \begin{vmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{vmatrix} \mathbf{E}$	$\mathbf{D} = \varepsilon(r, T, E) \cdot \mathbf{E}$
Conductivity Model (Ohm's Law)	$\mathbf{J} = \sigma \mathbf{E}$	$\mathbf{J} = \begin{vmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{vmatrix} \mathbf{E}$	$\mathbf{J} = \sigma(r, T, E) \cdot \mathbf{E}$
Magnetic Model (permittivity)	$\mathbf{B} = \mu \mathbf{H}$	$\mathbf{B} = \begin{vmatrix} \mu_{xx} & \mu_{xy} & \mu_{xz} \\ \mu_{yx} & \mu_{yy} & \mu_{yz} \\ \mu_{zx} & \mu_{zy} & \mu_{zz} \end{vmatrix} \mathbf{H}$	$\mathbf{B} = \mu(r, T, H, history) \cdot \mathbf{H}$

It is still not a full story. What more should be added?

2. For numerical solution field equation are often written with respect to potential, not field vectors

You already know a lot about potentials.

Examples are:

1. ***Scalar potential of electric field***: the potential $U(x, y, z)$ is such scalar quantity that $\mathbf{E} = -\text{grad}U$
   Question: Why we are sure that such potential really exists?
2. **Scalar potential of magnetic field**: the potential $U_M\,(x, y, z)$ is such scalar quantity that $\mathbf{H} = -\text{grad}U_M$
   Again: please think when the U_M exists and when it does not exists
3. **Vector magnetic potential**: the potential $\mathbf{A}(x, y, z)$ is such vector quantity that $\mathbf{B} = \text{rot}\mathbf{A}$
   Question: Does the vector potential $\mathbf{A}$ always exists?

Why the potential is numerically better than field vectors $\mathbf{E}, \mathbf{D}, \mathbf{H}, \mathbf{B}$?

Simplified set of Maxwell equations

Solving the full set of Maxwell equations is almost impossible (and always impractical)

Simplified set of Maxwell equations (examples):

1. **Electrostatics:**
   a) Nothing is changed in time;
   b) No magnetic field exists ($\mathbf{B}=0;\ \mathbf{H}=0$)
2. **Magnetostatics:**
   a) Nothing is changed in time;
   b) No electric field exists ($\mathbf{E}=0;\ \mathbf{D}=0$)
3. **AC Magnetics:**
   a) Every field quantity changed in time sinusoidal with the same frequency: $H = H_0 \cos(\omega t + \varphi_0)$
   b) No displacement current: $\mathbf{J}_\mathbf{D} = d\mathbf{D} / dt = 0$

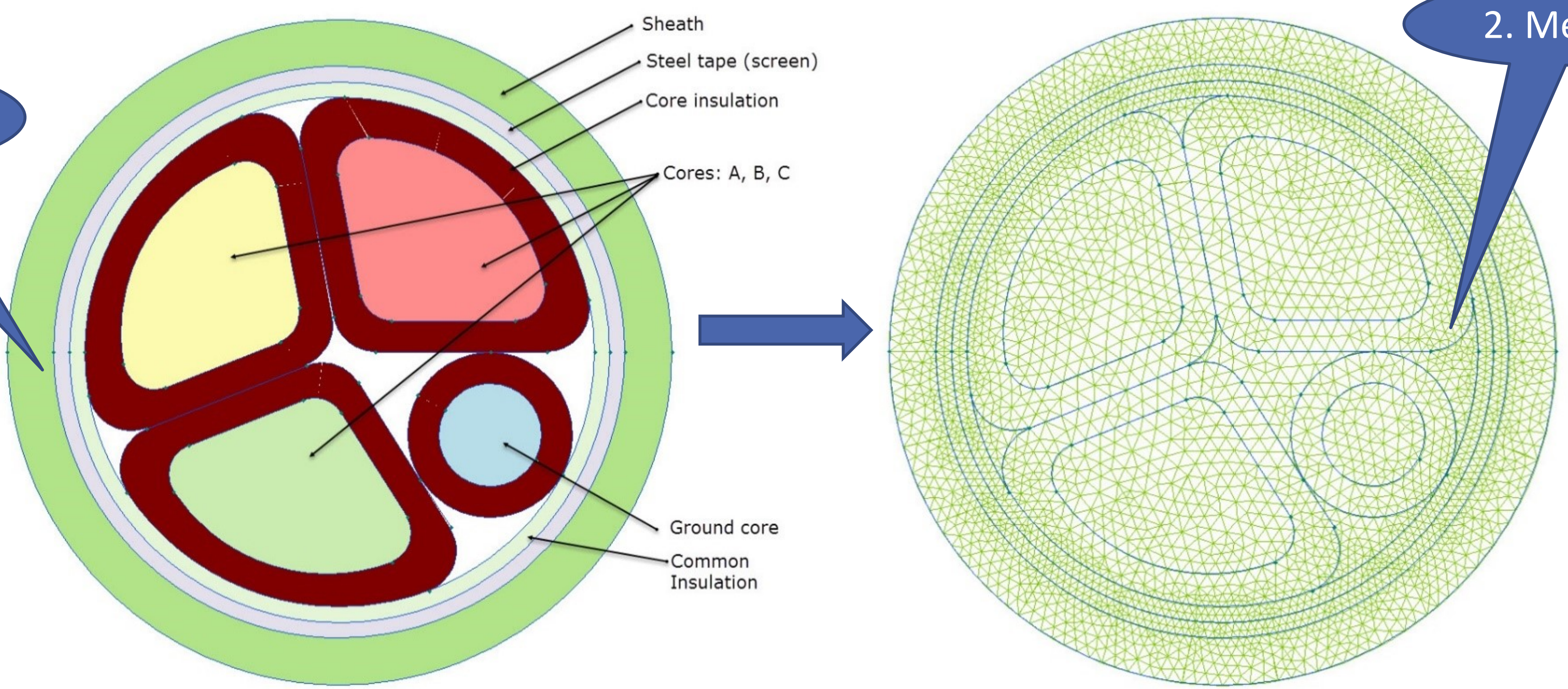
What’s more?

Geometry + Mesh + Boundary Conditions = Result

Every numerical problem is solved:

- In the given geometry (bounded or unbounded) – the geometry is always a simplification
- In discrete space (by the mesh)
- With given boundary conditions

1. Geometry



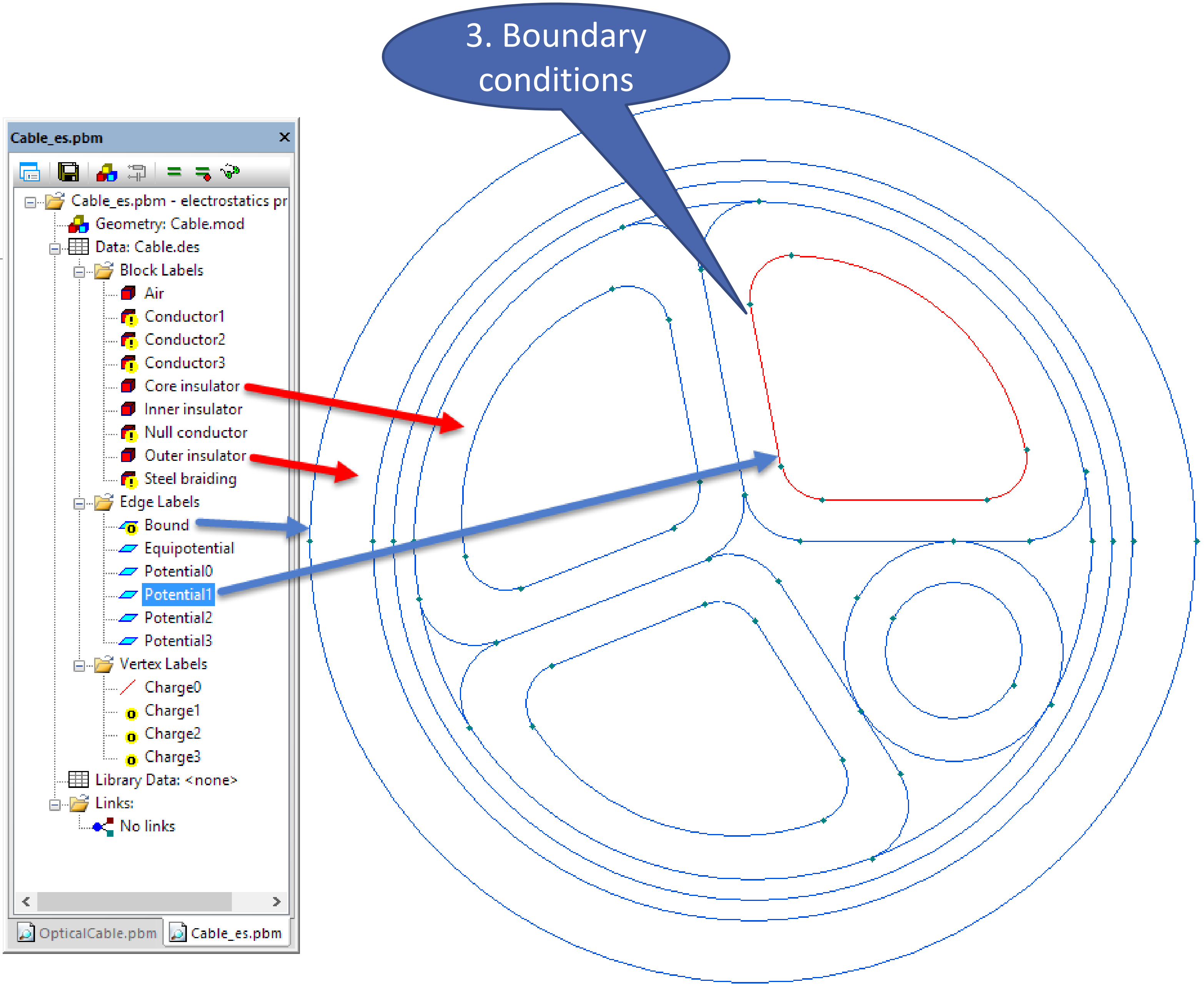
2. Mesh

What’s more?

Boundary Conditions & Field Sources

Each field problem is solved with boundary conditions (BC).

1. **BC reflects the influence of the world outside our model**
2. **Sometimes BC serve as a field source (e.g. given potential)**
3. **BC may reflect the symmetry condition when our model is ½ or ¼ of the real geometry**

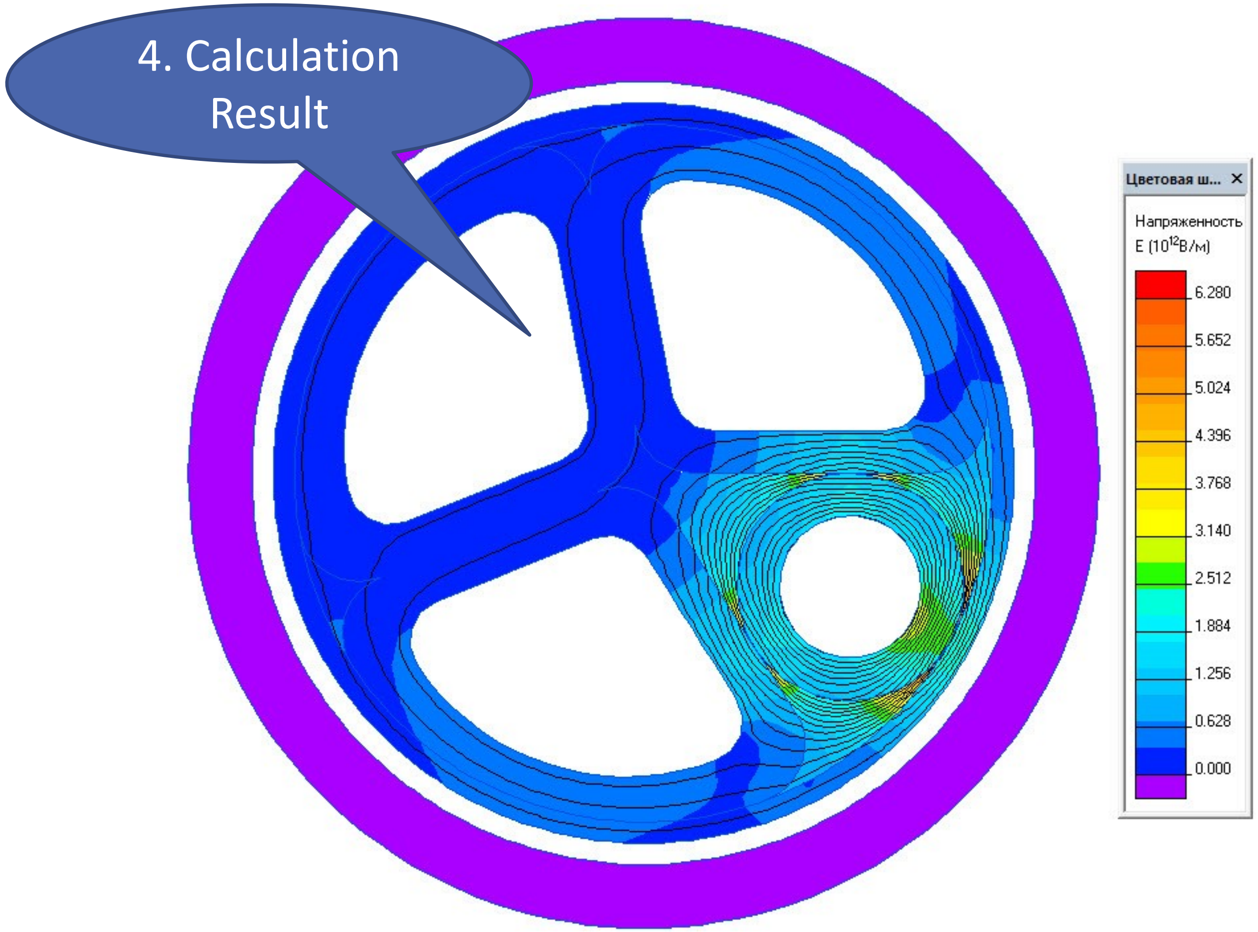


The purpose of computing is insight, not numbers

Richard Hamming, 1962

Typing is no substitute for thinking

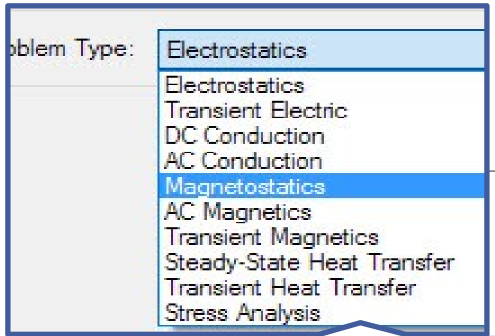
Richard Hamming, 1987



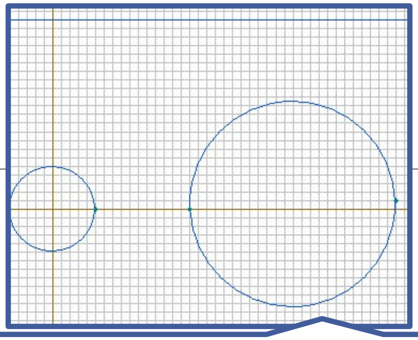
What we use to understand the calculated field:

1. Field pictures: color plot, vector plot, field lines
2. Local field values
3. Integral field quantities: flux, energy, force
4. XY-plot over various cross-section
5. Animated pictures

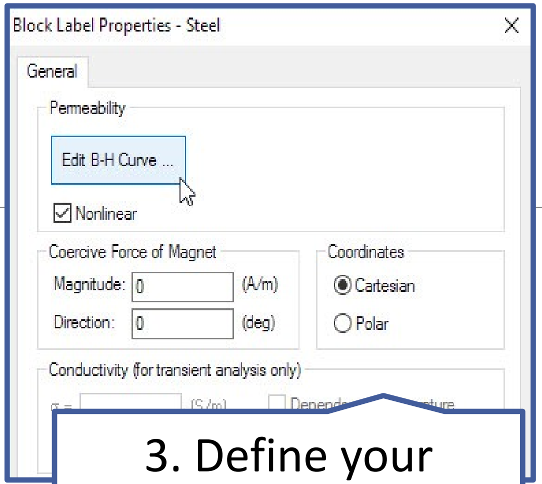
Resume: What you need to solve a field problem numerically?



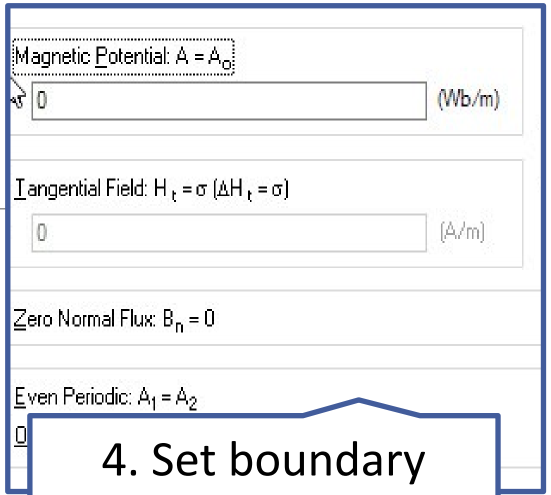
1. Choose formulation



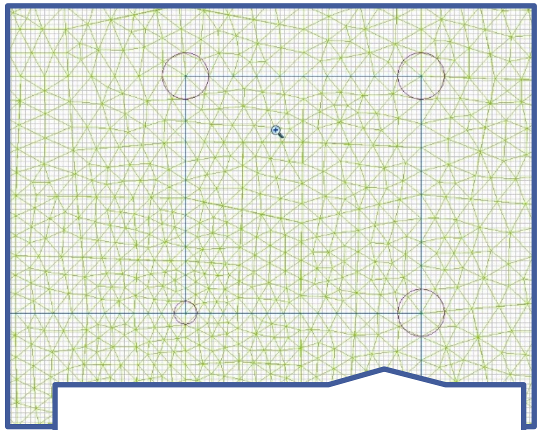
2. Draw your geometry



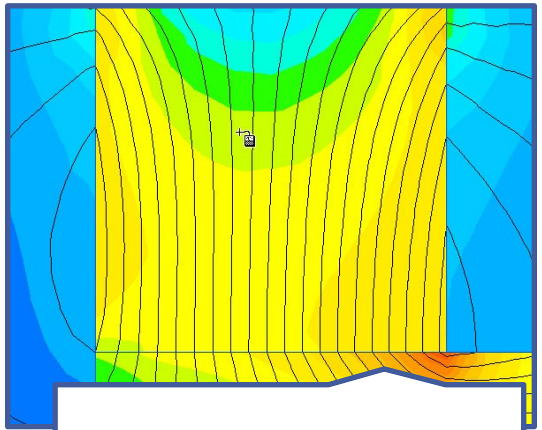
3. Define your materials



4. Set boundary conditions



5. Build the mesh



6. Enjoy your results

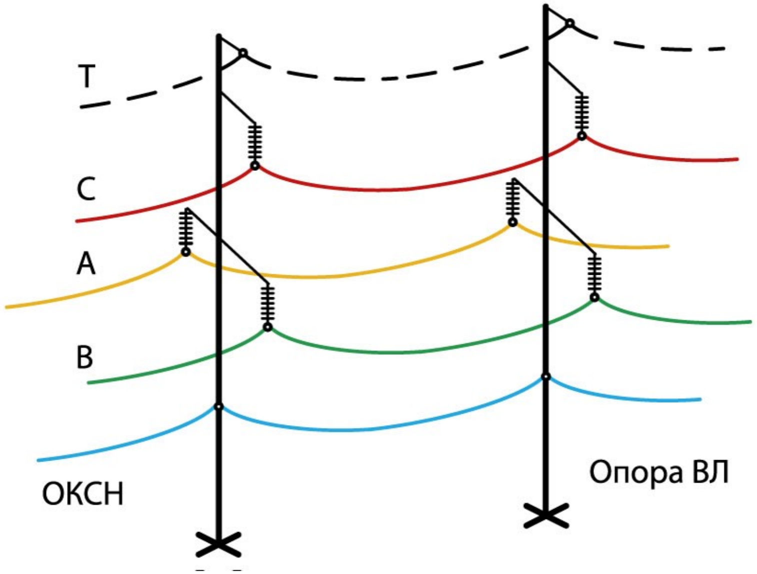
Examples:

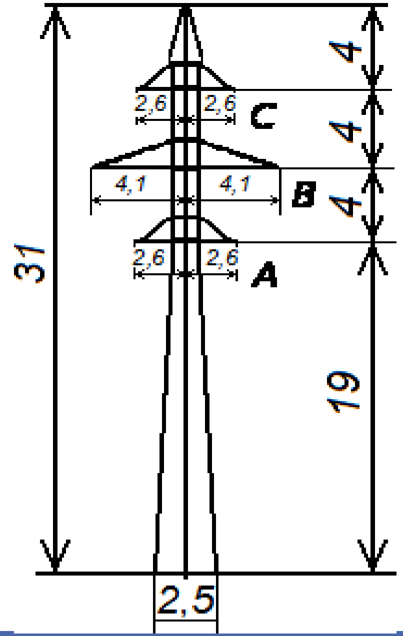
1. Determining the best place for optical cable in overhead power transmission line

The communication fiber-optical cable if often mounted along with high voltage overhead power line.

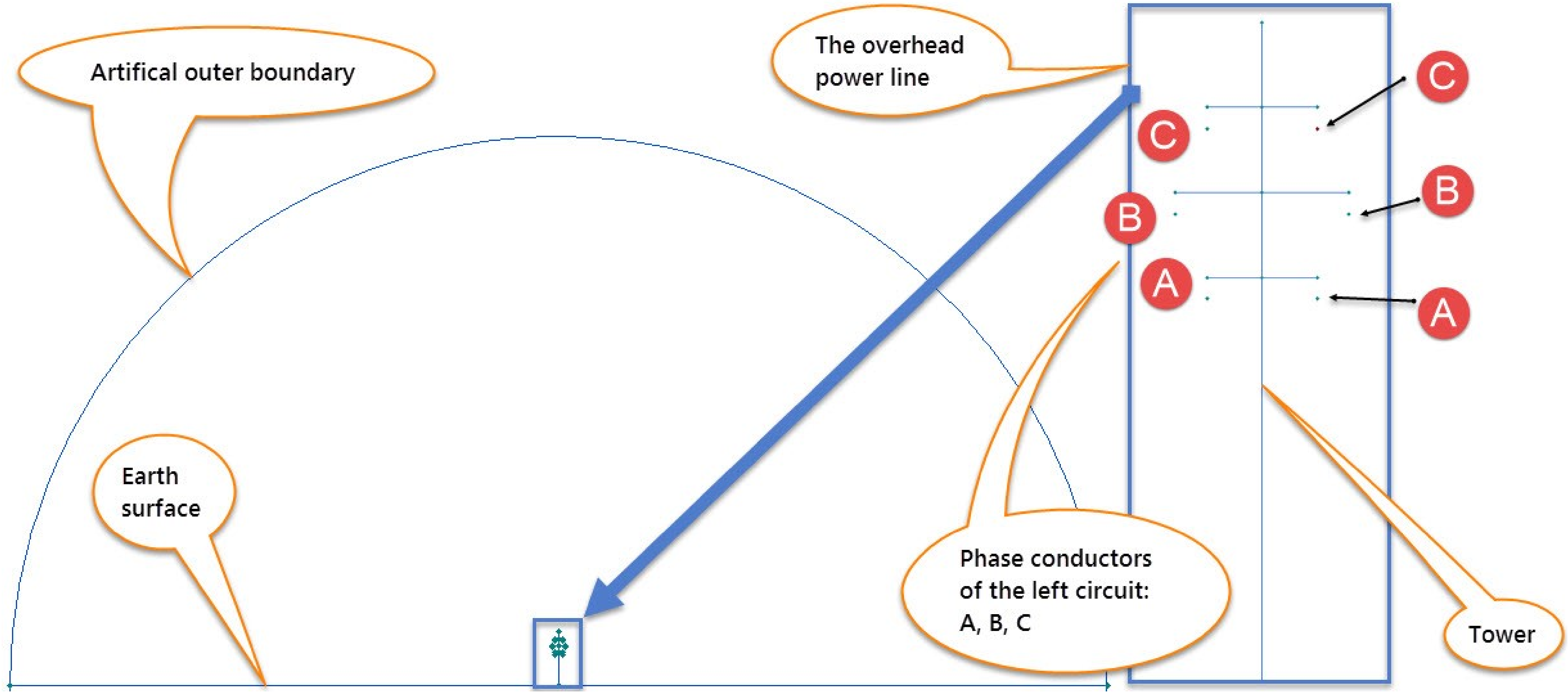
The cable itself is self-supported and fully dielectric, i.e. it contains no metal parts.
An optical cable should be properly located relative to phase conductors. What is a proper location?

According the Russian electrical code (ПУЭ) this is a place where the average potential U reaches its minimal value, $U <=$12 kV (25 kV with using special polyethylene). Also the electric filed $\mathbf{E}$ should be checked.





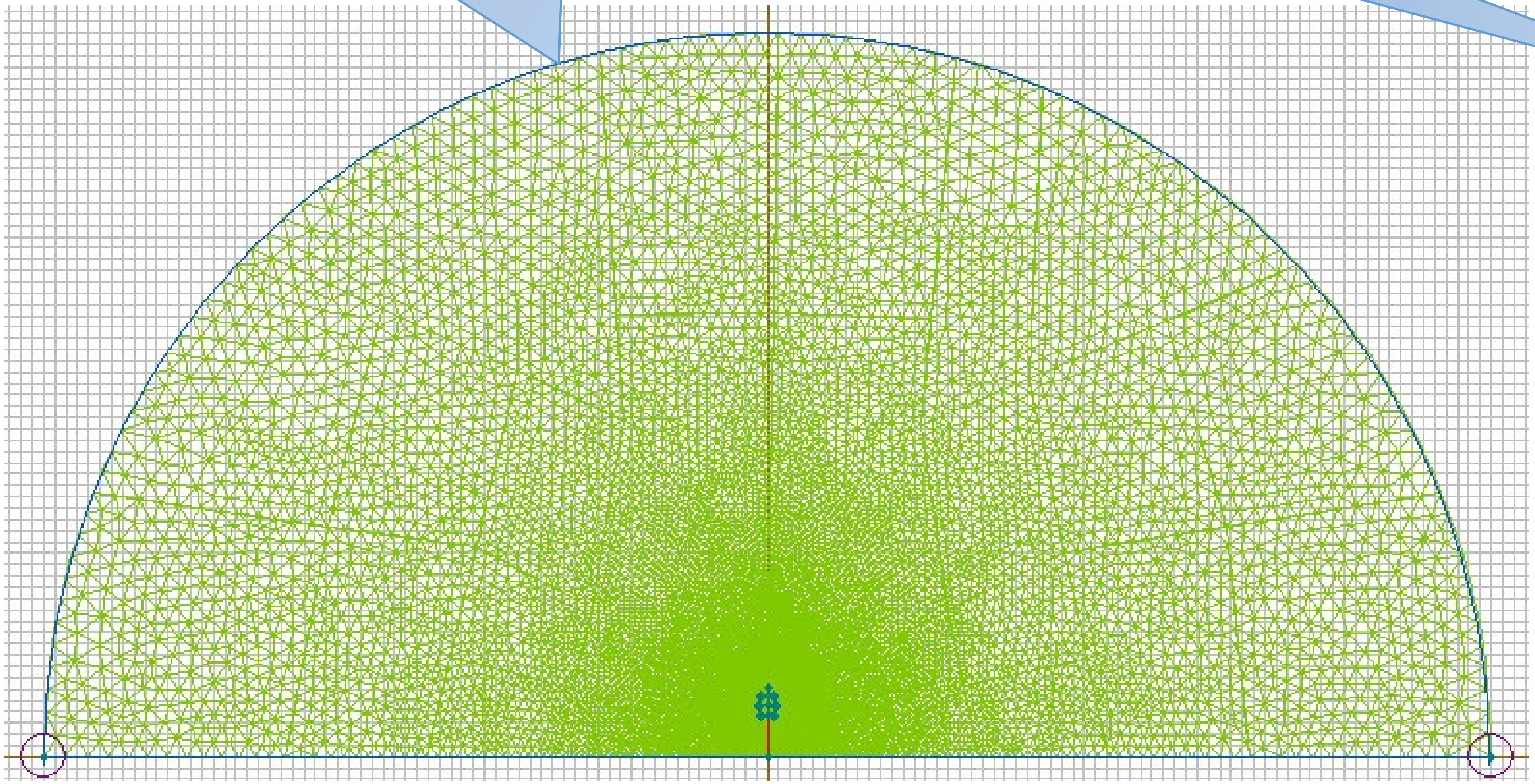
Model of Overhead Line: the cross-section

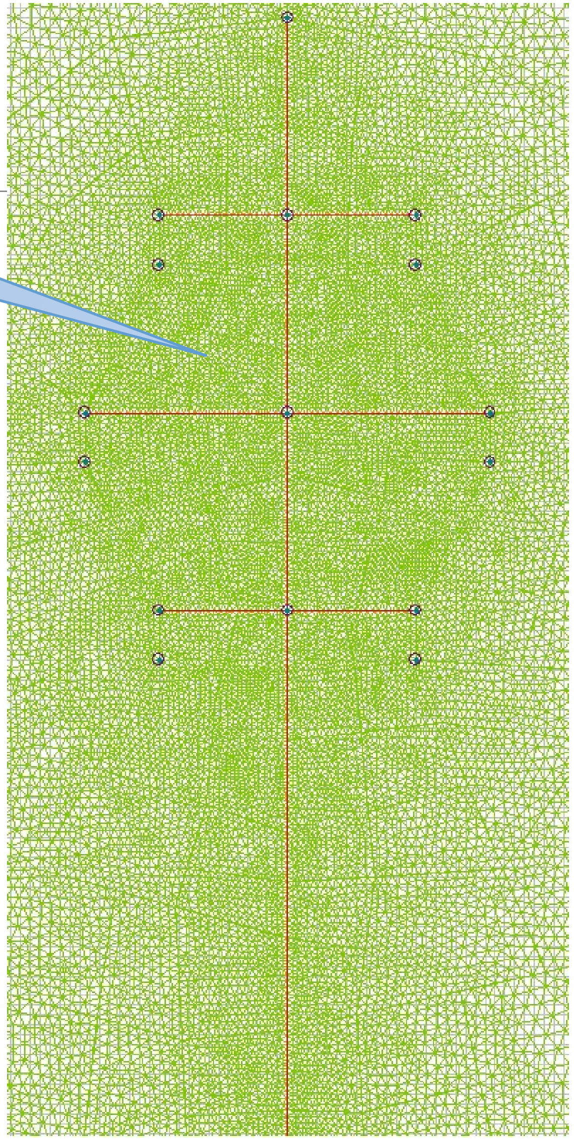


Model of Overhead Line: the geometry model

The whole model

The tower

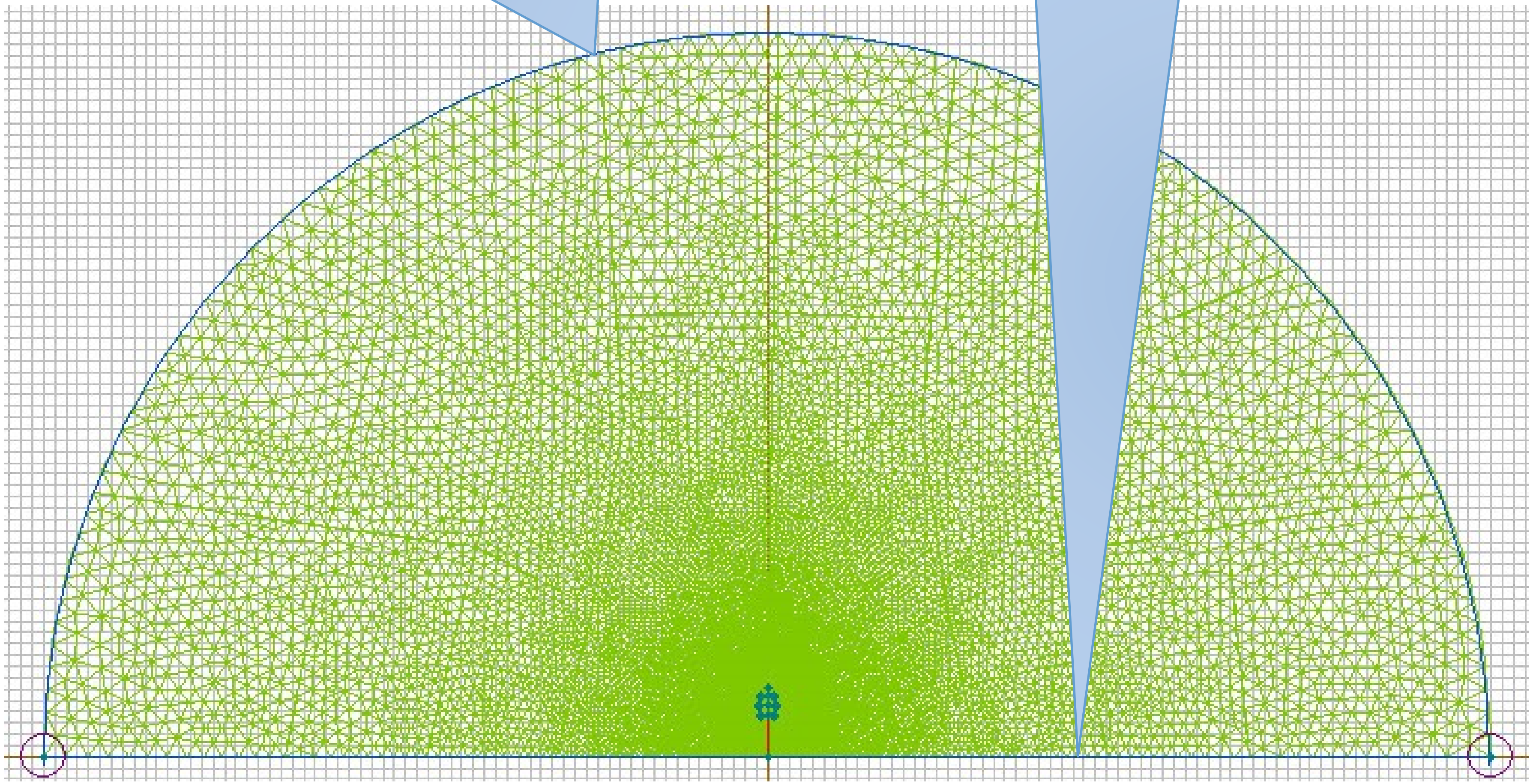


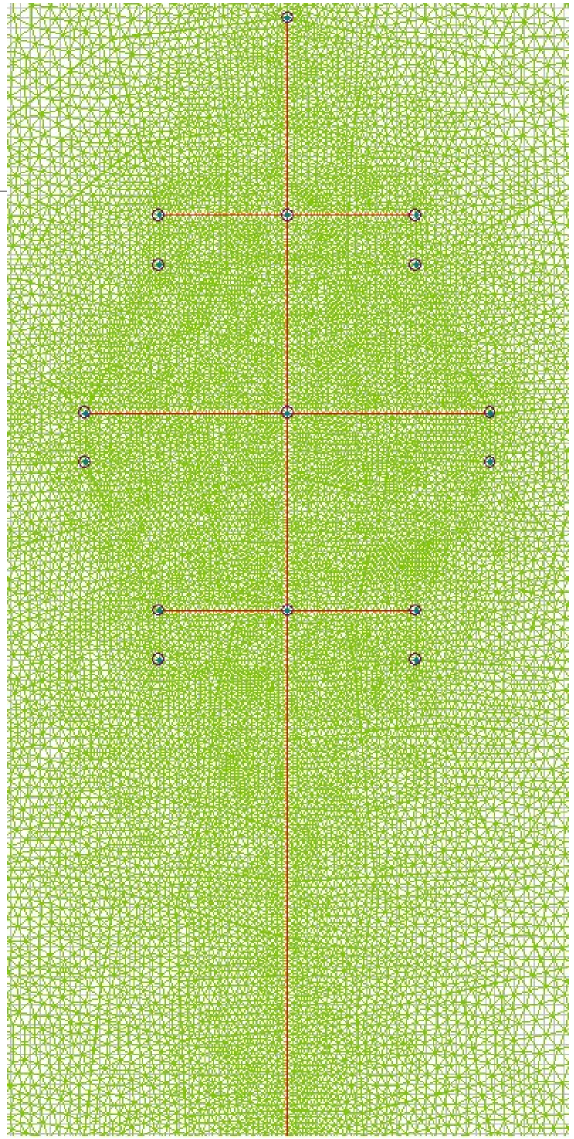


Model of Overhead Line: boundary condition

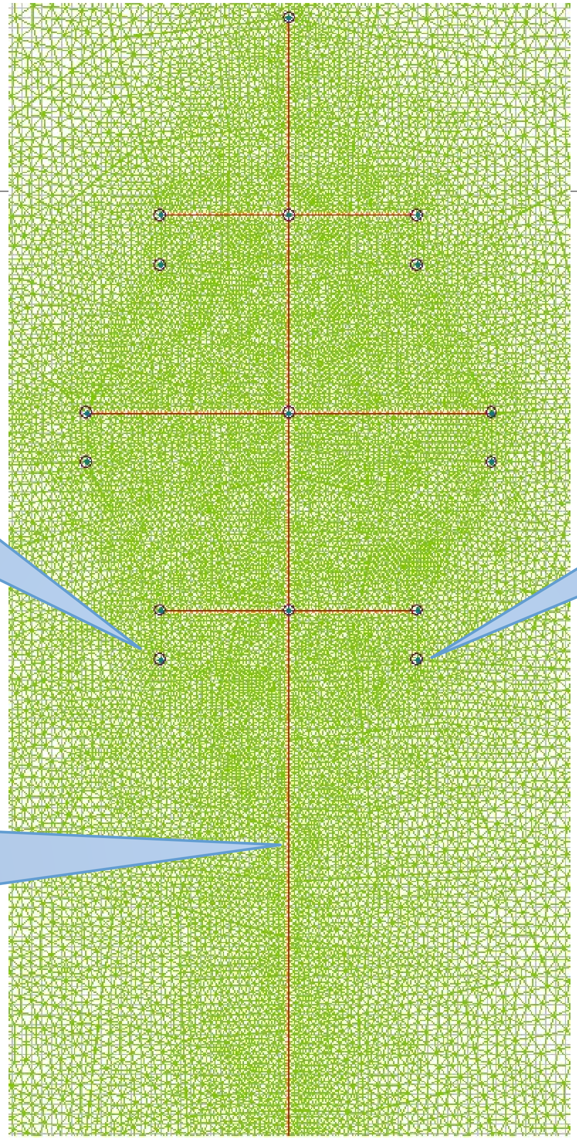
Outer boundary: no field here (U=0)

Earth surface: U=0





Model of Overhead Line: boundary condition

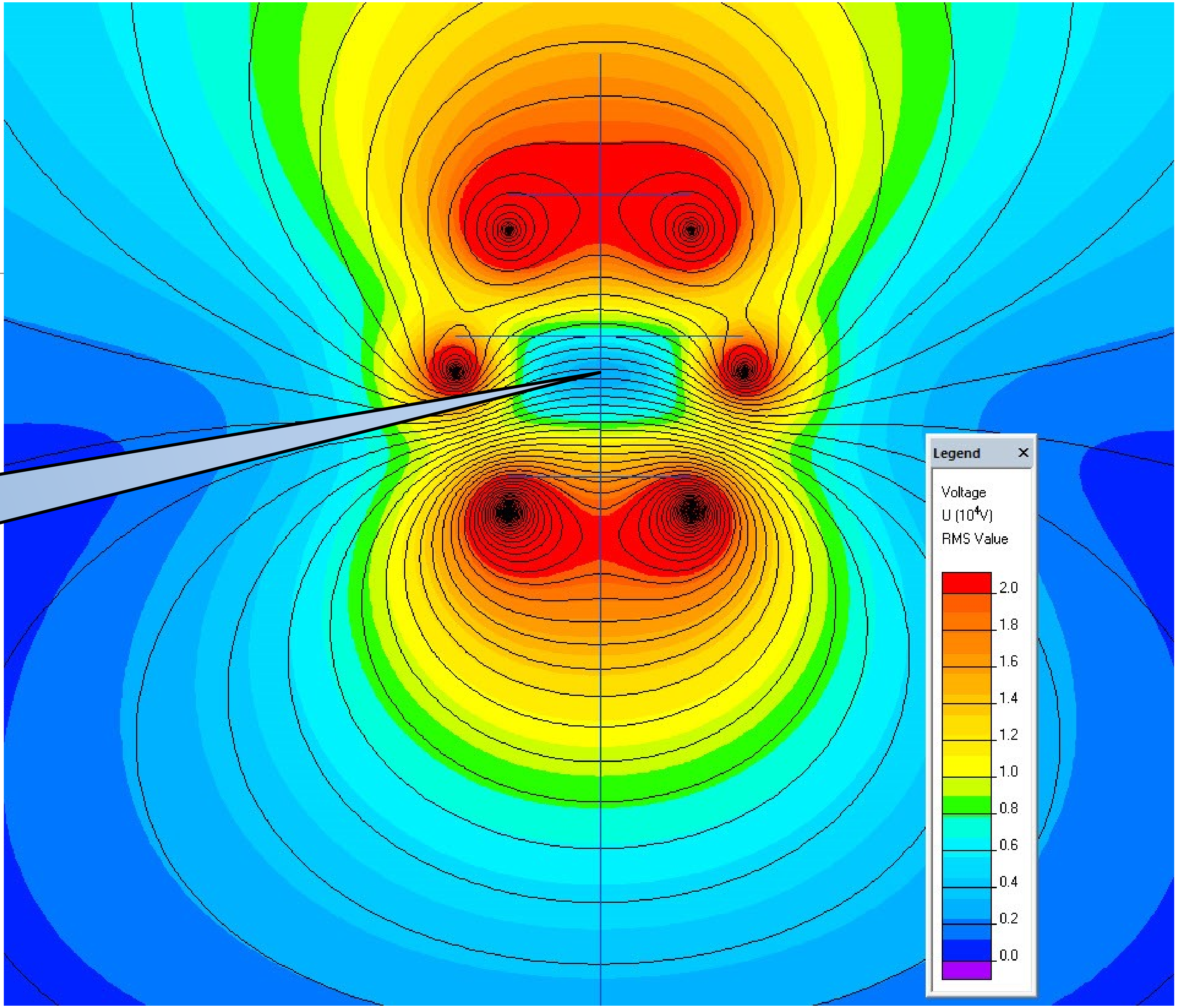


Left Circuit:
Momentary phase voltages
U_{A1}, U_{B1}, U_{C1}

Right Circuit:
Momentary phase voltages
U_{A2}, U_{B2}, U_{C2}

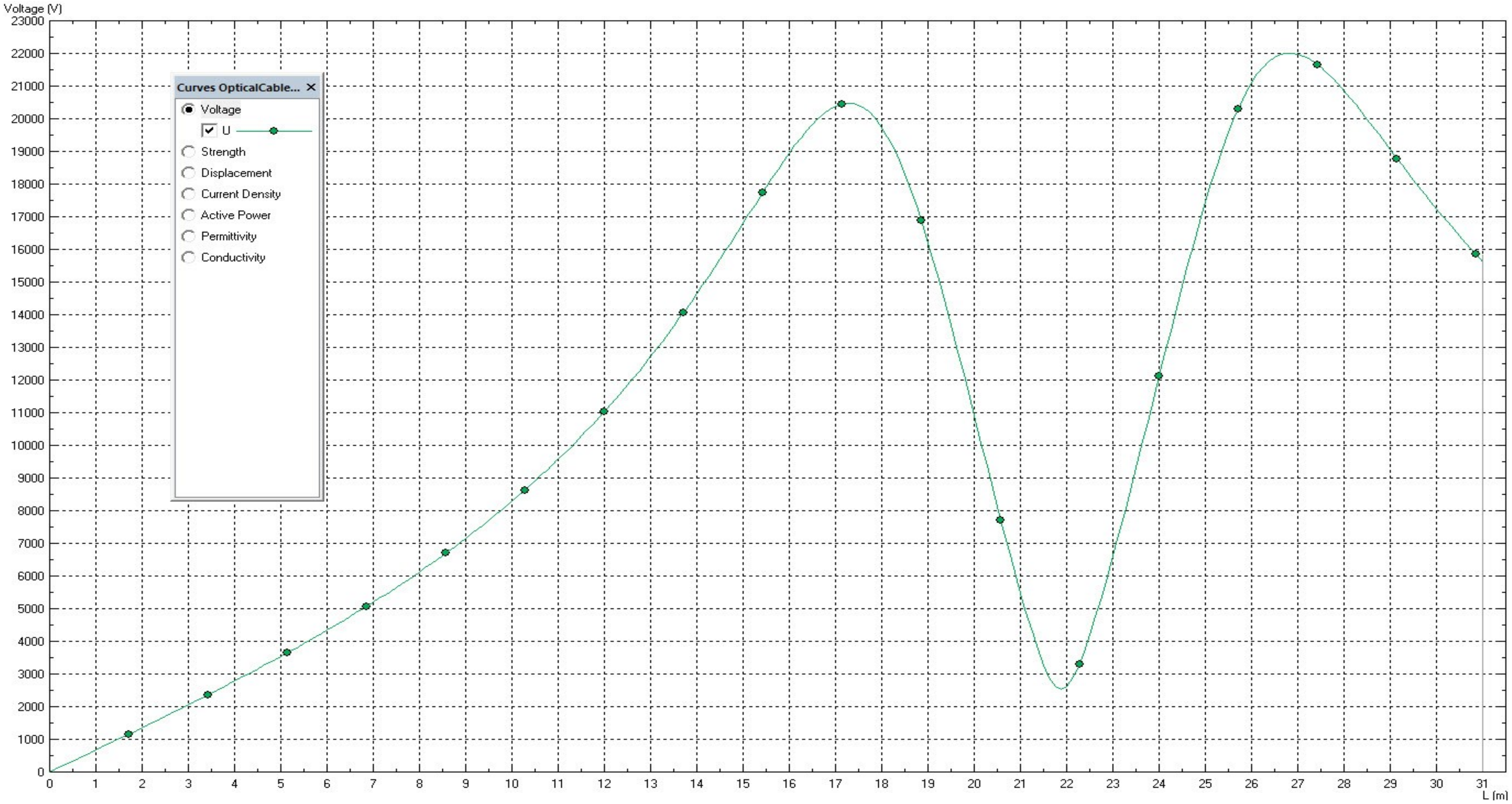
The Tower:
Is grounded: $U=0$

Model of Overhead Line: the solution

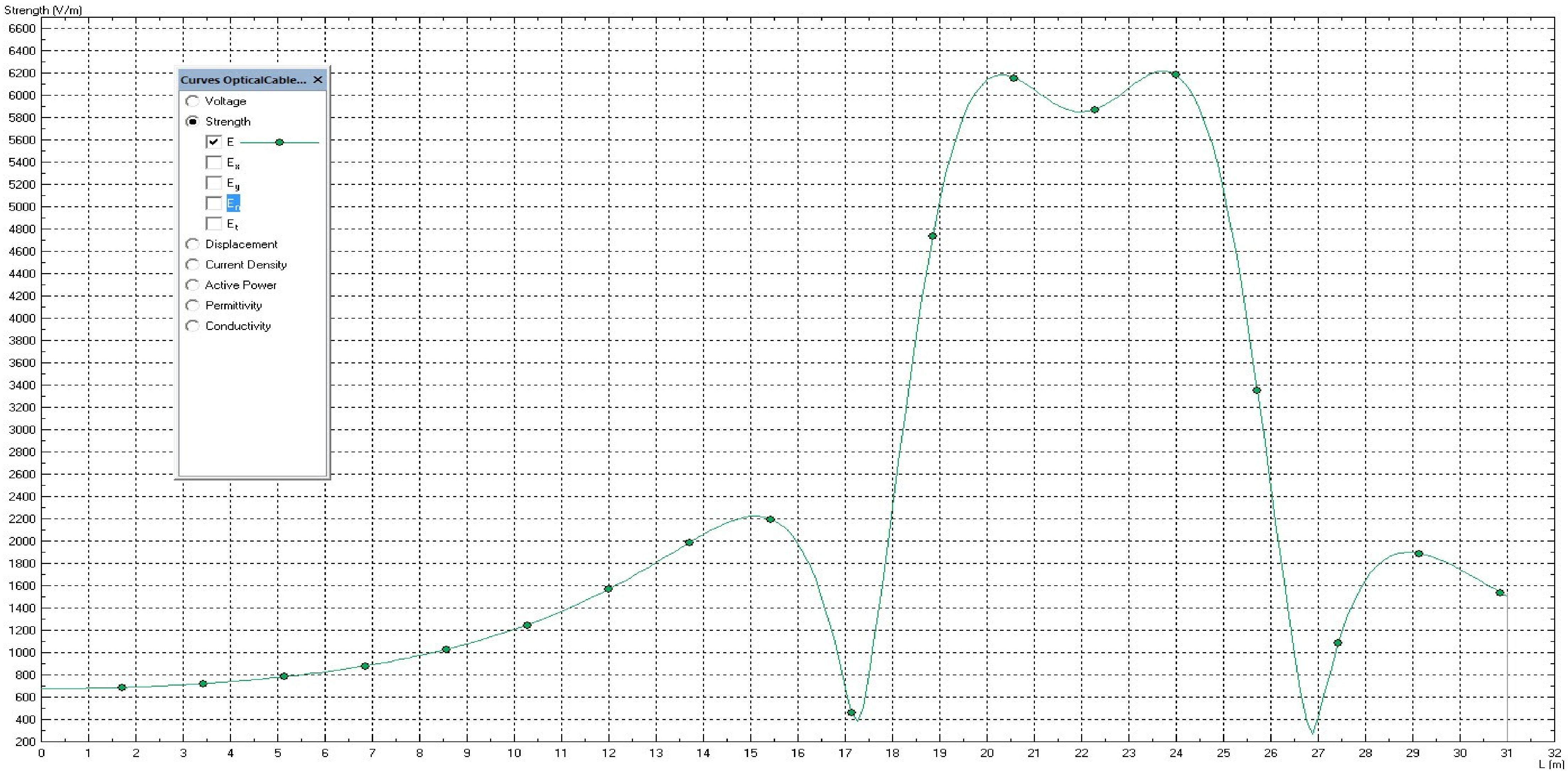


Good place for an optical cable (the potential U<6 kV)

Model of Overhead Line: XY-plot of Potential *U(y)*



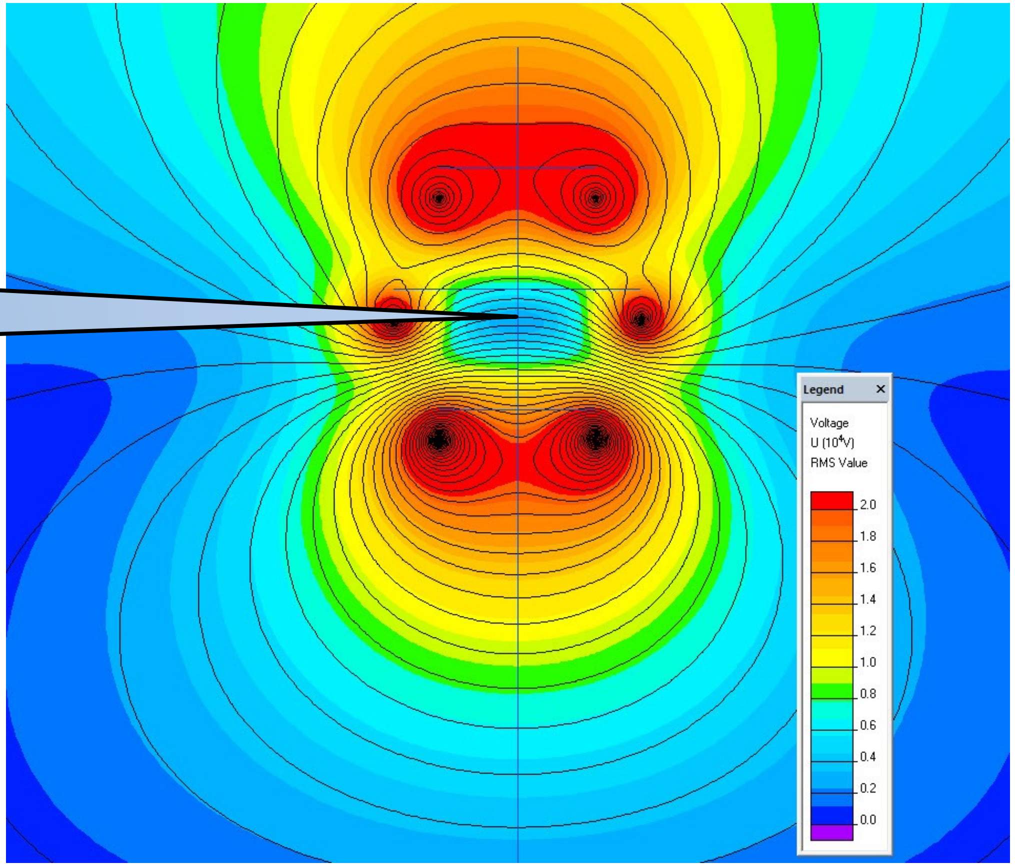
Model of Overhead Line: XY-plot of electric field $E(y)$



Model of Overhead Line: Conclusion

By numerical simulation we have found a good place for the self-supported dielectric optical cable on the tower:

Good place for an optical cable



Optimal coordinates are: $x=0; y=22$ m

The potential is this point is: $U=2.7\ kV$

The electric field stress there is: $E=6.0\ kV/m$

What Next?

Next week we will focus on Electrostatic analysis – the simplest case of EM fields.

We will learn how to:

1. Create a QuickField problem,
2. Draw the geometry
3. Set up boundary conditions
4. Manage the mesh density
5. Obtain the problem solution, and
6. Understand and analyze the results

Be prepared: download and install your QuickField Student Edition from: http://www.quickfield.com/free_soft.htm