AC Magnetic Field with Eddy Currents

1. What happens when field gets changing with time?
2. Solid and stranded conductors
3. The governing equation.
4. Phasor notation
5. How to take into account steel saturation
6. Calculated field Quantities: locals and integrals
7. Boundary conditions: Dirichlet, Neumann.

   Example: Magnetic shielding with AC

   Example: rotating magnetic field

What happens when time is added?

The same as DC

Gauss' theorem for magnetic field:	$\mathrm{div}\mathbf{B} = 0$	Vector magnetic potential exists: $\mathbf{A}: \mathbf{B} = \mathrm{rot}\mathbf{A}$ $\oplus$ Calibration: $\mathrm{div}\mathbf{A} = 0$
Ampere Law:	$\mathrm{rot}\mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$	We neglect the displacement current: $\frac{\partial \mathbf{D}}{\partial t} = 0$

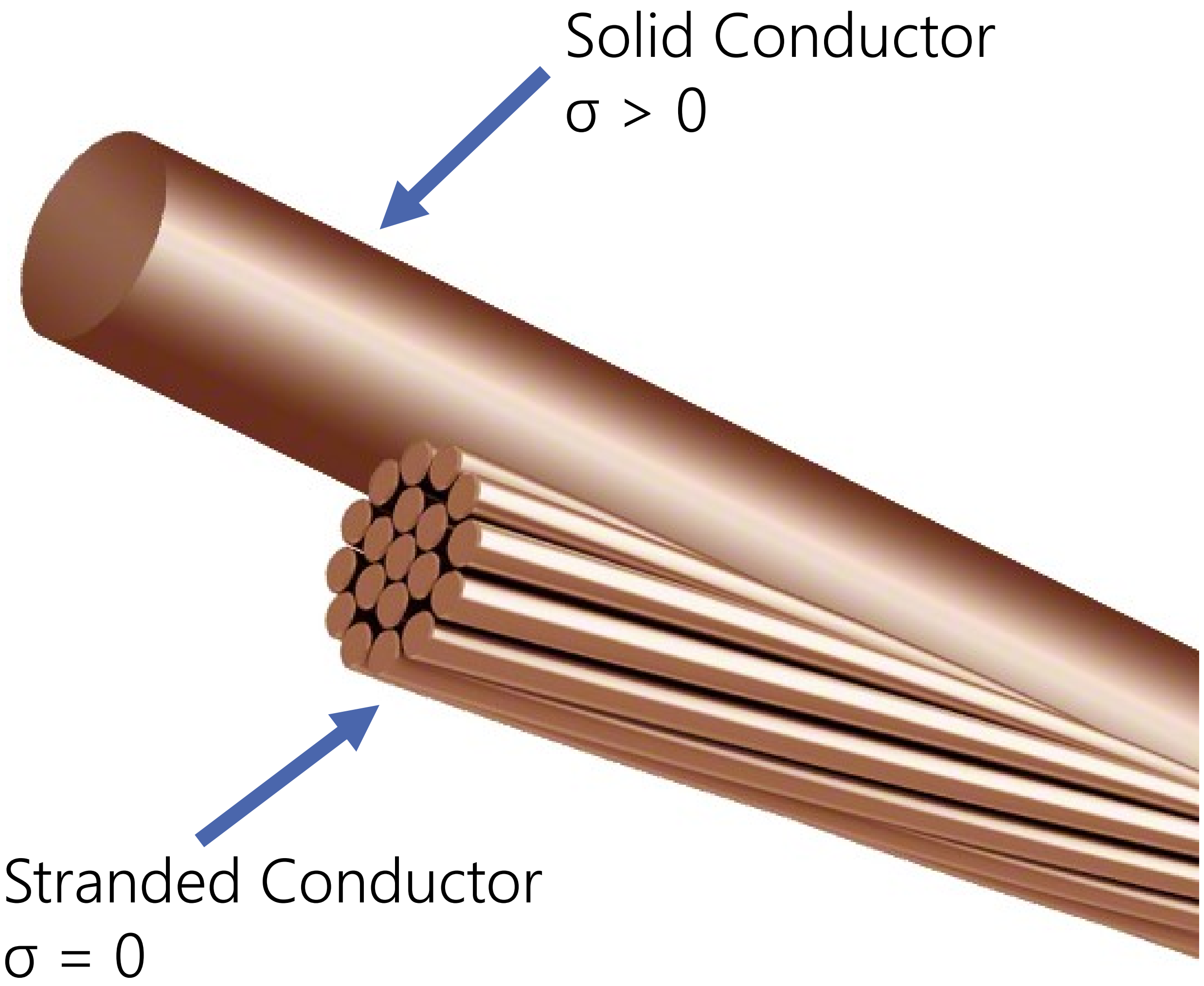
New behavior

Faraday's Law of induction:	$\mathrm{rot}\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	Eddy electric field exists. In conductors it creates eddy currents
Electric Field contains two components	1) $\mathbf{E}_{potential} = \frac{\Delta U}{L_Z}$ 2) $\mathbf{E}_{eddy} = \frac{\partial \mathbf{A}}{\partial t}$	

The same with phasor notation

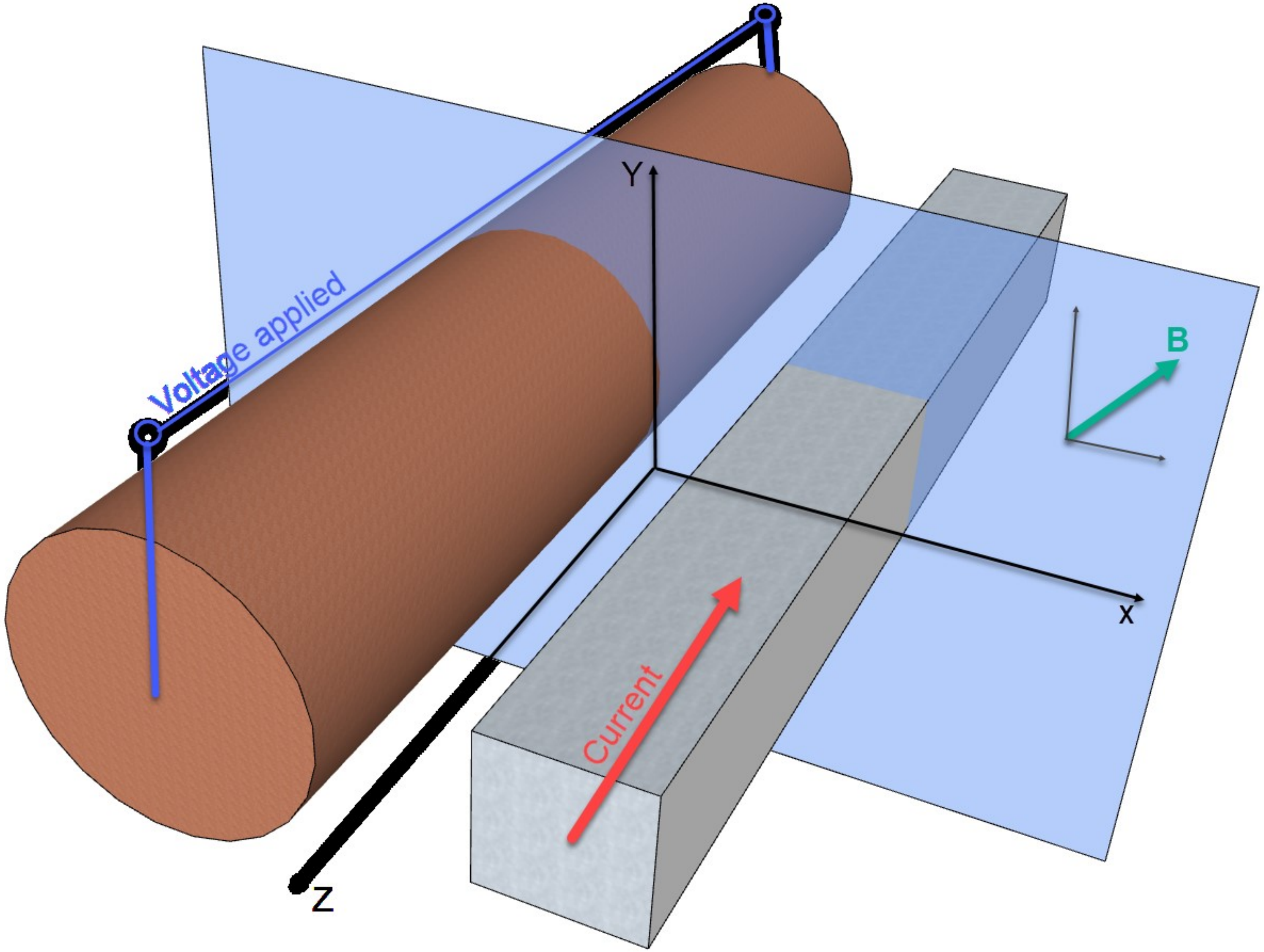
Gauss' theorem for magnetic field:	$\mathrm{div}\dot{\mathbf{B}} = 0$	Vector magnetic potential exists: $\dot{\mathbf{A}}: \dot{\mathbf{B}} = \mathrm{rot}\dot{\mathbf{A}}$ $\oplus$ Calibration: $\mathrm{div}\dot{\mathbf{A}} = 0$
Ampere Law:	$\mathrm{rot}\dot{\mathbf{H}} = \dot{\mathbf{J}}$	We neglect the displacement current:
Faraday's Law of induction:	$\dot{\mathbf{E}}_{eddy} = -j\omega\dot{\mathbf{A}}$	Eddy electric field exists. In conductors it creates eddy currents
Electric Field contains two components:	1) $\dot{\mathbf{E}}_{potential} = \dfrac{\Delta\dot{U}}{L_Z}$ 2) $\dot{\mathbf{E}}_{eddy} = j\omega\dot{\mathbf{A}}$	

Solid and Stranded Conductors



	Solid	Stranded
Conductivity	Yes	No
External Current	Yes	Yes
Eddy Current	Yes	No
Applied Voltage	Yes	No
Included to circuit	Yes	No

Magnetic field in plane, Current – out of plane



	Solid	Stranded
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Governing Equation

$$\mathrm{rot}(\frac{1}{\mu}\mathrm{rot}\dot{\mathbf{A}}) - j\omega\dot{\mathbf{A}} = -\dot{\mathbf{J}}_{source} \qquad \mu = f(\mathrm{rot}\mathbf{A})$$

Poisson equation
(coordinate independent form)

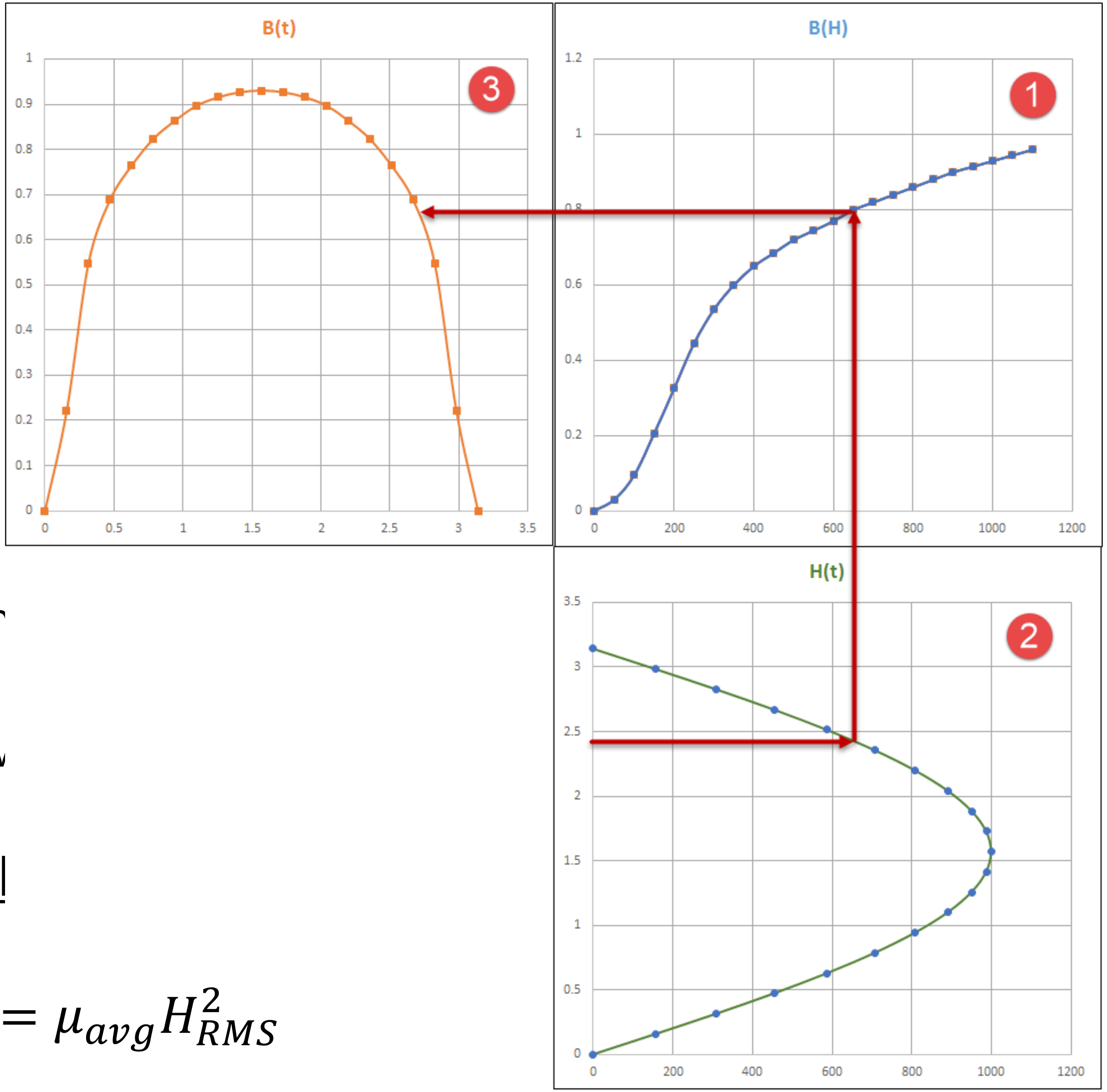
$$\frac{\partial}{\partial x}\left(\frac{1}{\mu_y}\frac{\partial \dot{A}}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{1}{\mu_x}\frac{\partial \dot{A}}{\partial y}\right) - j\omega\dot{A} = -J_{source}$$

In Cartesian coordinates (x, y)

$$\frac{\partial}{\partial r}\left(\frac{1}{r\mu_z}\frac{\partial\left(r\dot{A}\right)}{\partial r}\right) + \frac{\partial}{\partial z}\left(\frac{1}{r\mu_r}\frac{\partial\left(r\dot{A}\right)}{\partial z}\right) - j\omega\dot{A} = -J_{source}$$

In cylindrical coordinates (r, z)

Saturated material



Let's assume H(t) is sine-wave (*plot 2*).

Then get B(t) (*plot 3*) curve through knowr

The resulting B(t) appears to be non sine-v

We decide choosing μ_{avg} in such a way to |

the time-average energy: $$\int_0^T H(t)B(t)\,dt = \mu_{avg} H_{RMS}^2$$

Magnetic boundary conditions

$$\dot{A} = f(x, y)$$

Plane

$$r\dot{A} = f(x, y)$$

Axisymmetric

Dirichlet Condition:
the known magnetic potential A (or rA)
(may depend on coordinates)

$$\dot{H}_t = \frac{1}{\mu}\frac{\partial \dot{A}}{\partial n} = \dot{\sigma}(x, y)$$

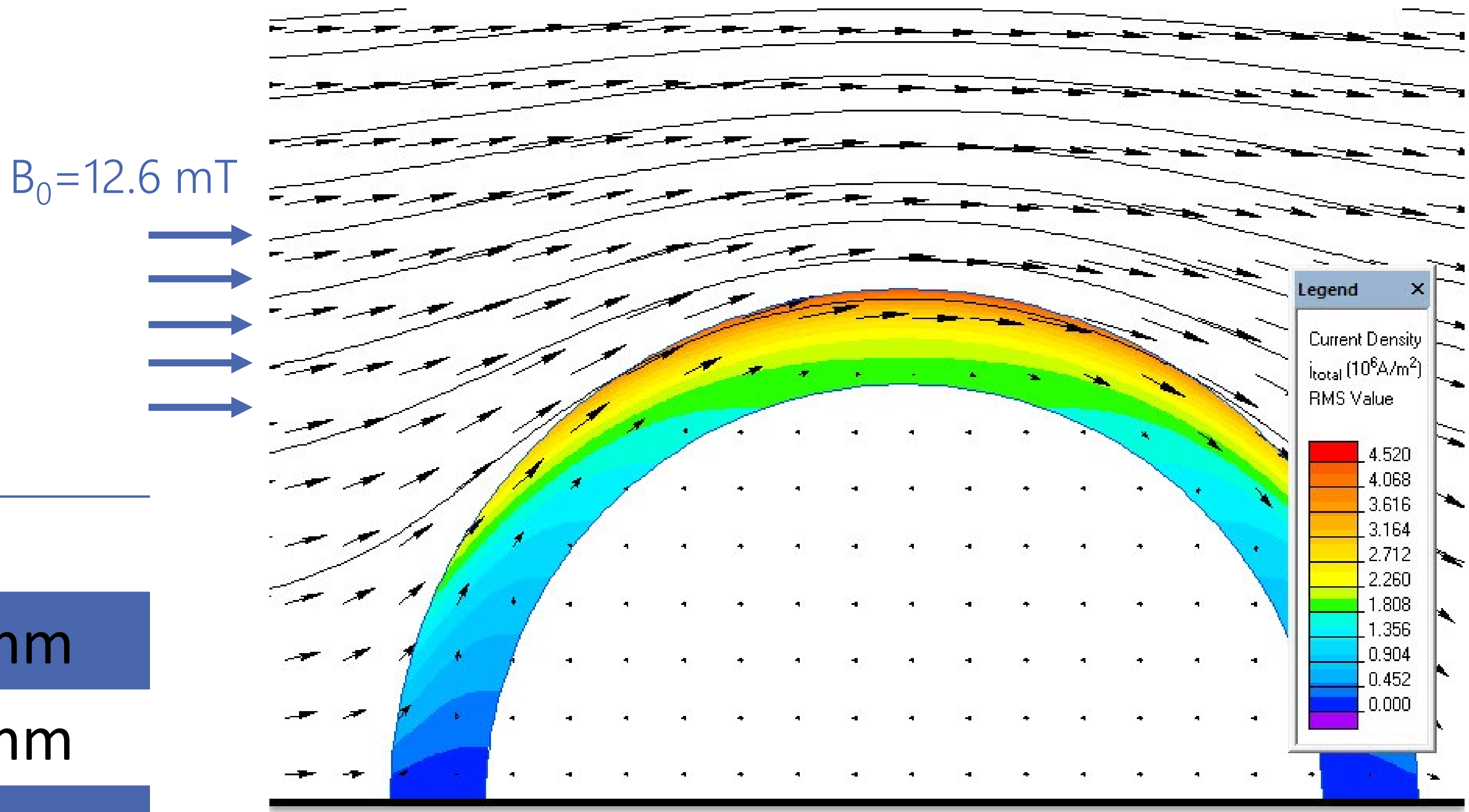
Neumann condition:
the known tangential H-field,
equal to surface current density

$$\dot{A} = const \Leftrightarrow \dot{B}_n = 0$$

Zero normal flux (superconductor):
A special type of Dirichlet condition

Example 1: AC Magnetic Shielding

Conductive spherical shield in uniform magnetic field B_0



Sizing:

External diameter	54 mm
Internal diameter	44 mm
Outer diameter	*500 mm*

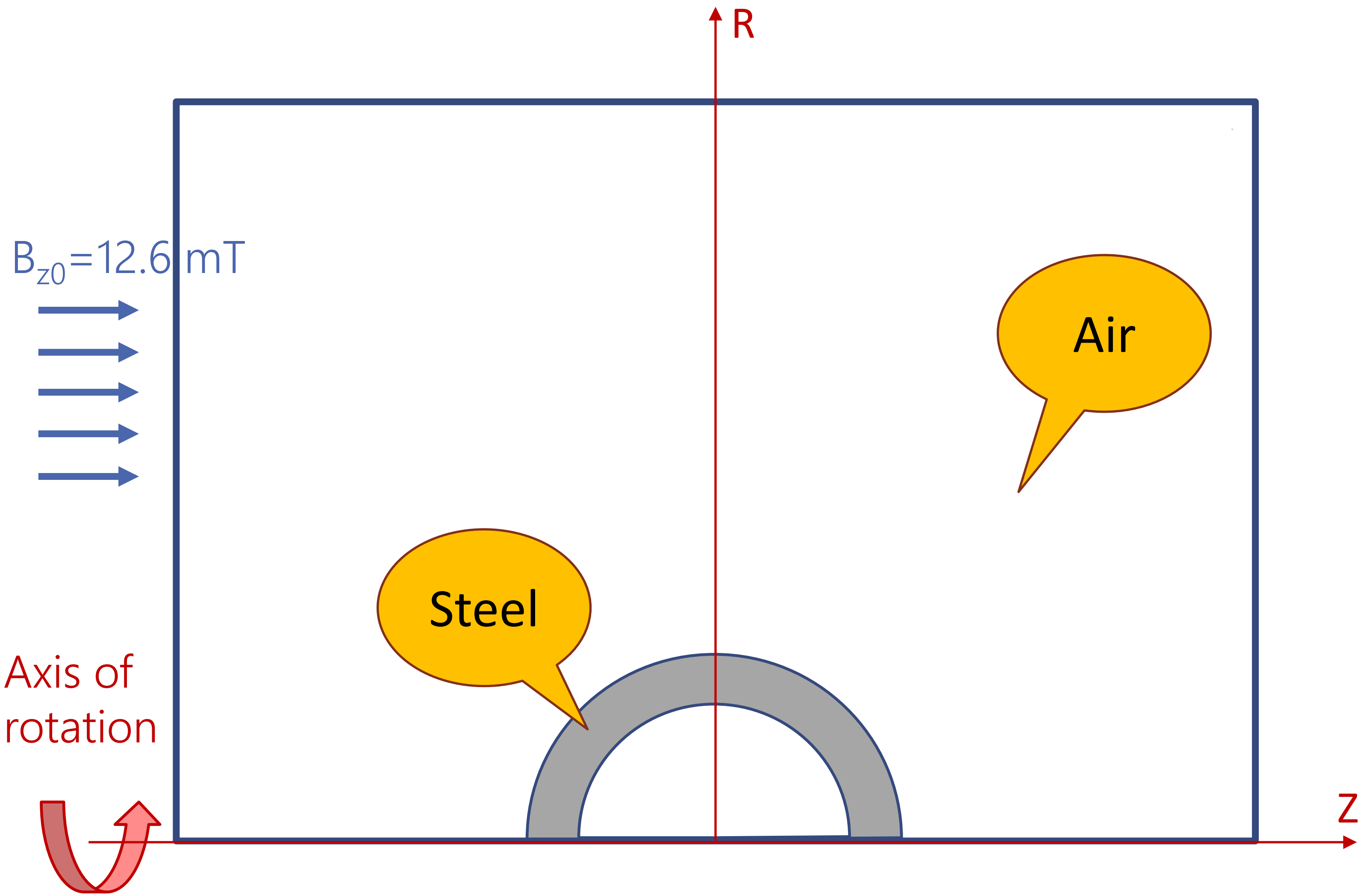
Z-uniform field boundary condition

$$\mathbf{B} = \mathrm{rot}\mathbf{A} = \begin{cases} B_Z = \dfrac{1}{r}\dfrac{\partial(rA)}{\partial r} \\ B_R = -\dfrac{1}{r}\dfrac{\partial(rA)}{\partial z} \end{cases}$$

Choose as: $rA = 0.5r^2 B_{Z0}$

Check it by differentiating:

$$B_Z = \frac{1}{r}\frac{\partial\left(\frac{1}{2}r^2 B_{Z0}\right)}{\partial r} = B_{Z0}$$

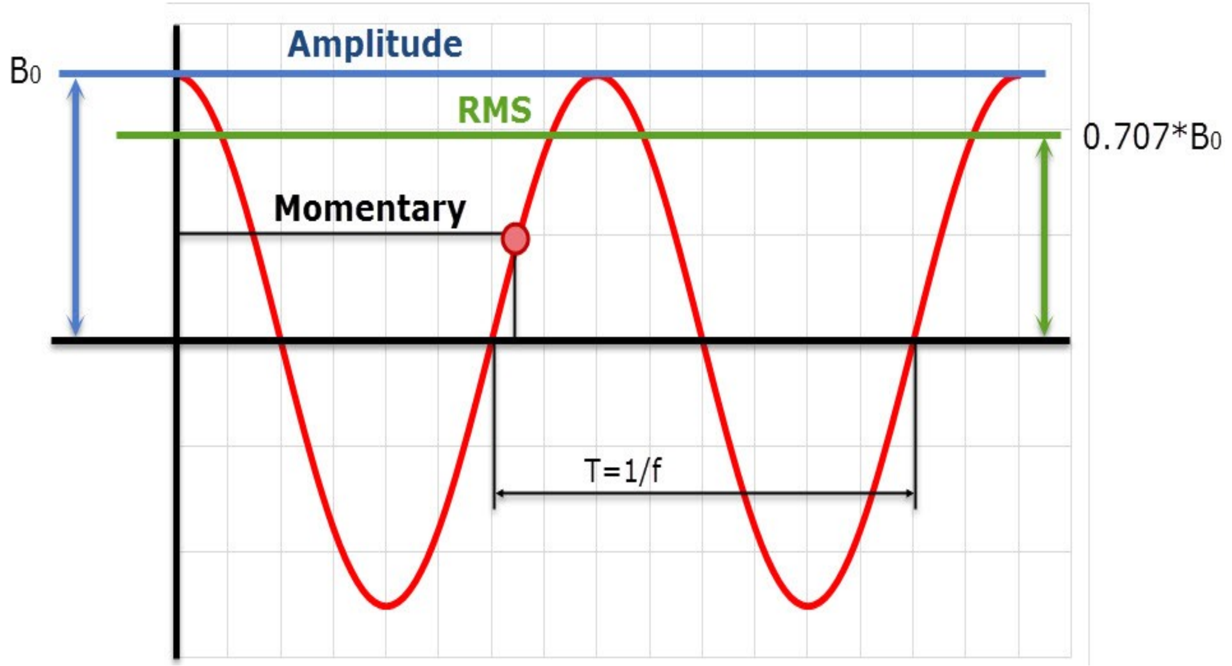


Calculation results

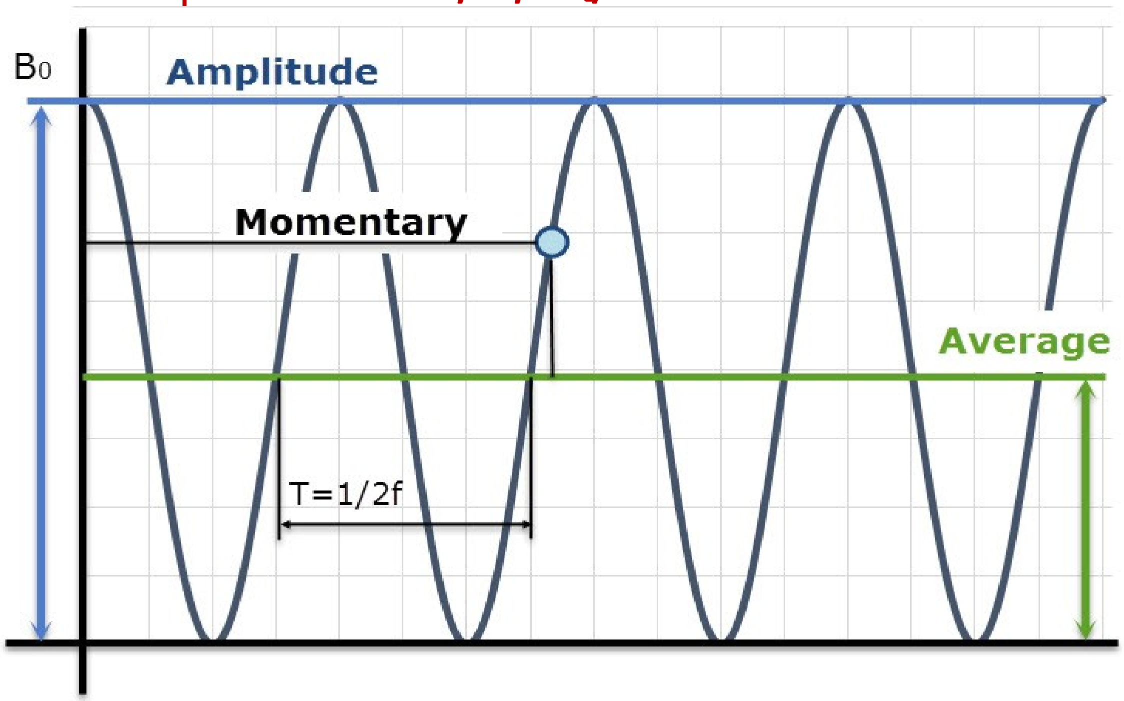
Each quantity is presented in one of 3 forms:

1. Peak value,
2. RMS value, or
3. Momentary value (by given φ)

Phasors: **B, H, J, A**



Non phasors: **S, F, Q, W**



Calculation results

1. Locals

Flux Density: $\dot{\mathbf{B}} = \mathrm{rot}\dot{\mathbf{A}}$

Field Strength: $\dot{\mathbf{H}} = \dfrac{1}{\mu}\dot{\mathbf{B}}$

Current Density: $\dot{\mathbf{J}}_{eddy} = j\omega\sigma\dot{\mathbf{A}}$

$$\dot{\mathbf{J}}_{extern} = \frac{\Delta\dot{U}}{L_Z}$$

$$\dot{\mathbf{J}}_{total} = \dot{\mathbf{J}}_{eddy} + \dot{\mathbf{J}}_{extern}$$

Power Loss: $q = \dfrac{1}{\sigma}\dot{\mathbf{J}}^2$

Energy density: $w = \dfrac{1}{2}\dot{\mathbf{B}} \cdot \dot{\mathbf{H}}$

Pointing Vector: $\dot{\mathbf{S}} = \dot{\mathbf{E}} \times \dot{\mathbf{H}}$

Calculation results

2. Integrals

Full Current: $$I = \int_S \mathbf{J} \cdot \mathbf{d}s$$

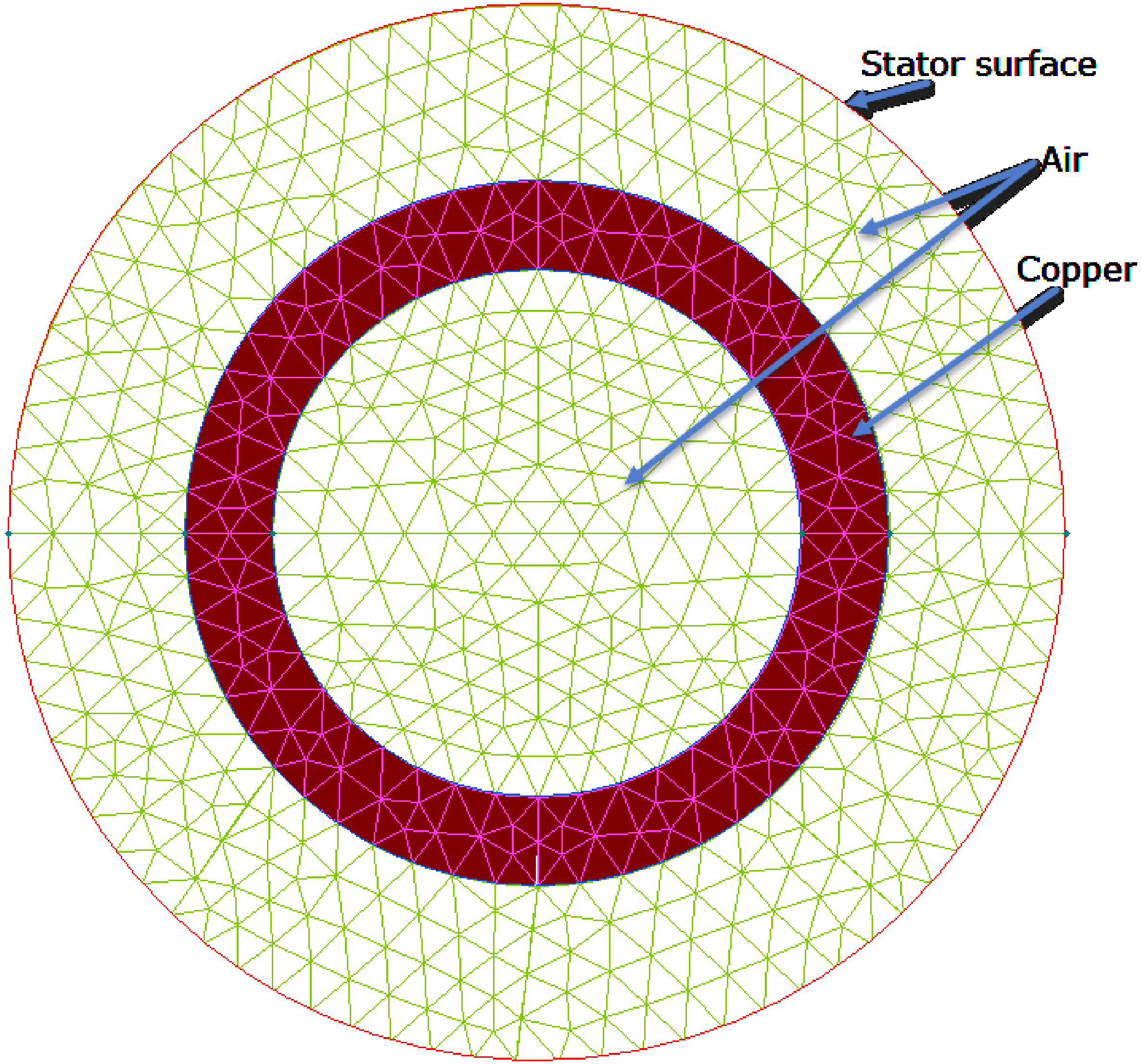
Mechanical force: $$\mathbf{F} = \frac{1}{2}\oint_S \big(\mathbf{H}(\mathbf{B}\cdot\mathbf{n}) + \mathbf{B}(\mathbf{H}\cdot\mathbf{n}) - \mathbf{n}(\mathbf{H}\cdot\mathbf{B})\big)\,ds$$

Mechanical torque: $$\mathbf{T} = \frac{1}{2}\oint_S \big((\mathbf{r}\times\mathbf{H})(\mathbf{B}\cdot\mathbf{n}) + (\mathbf{r}\times\mathbf{B})(\mathbf{H}\cdot\mathbf{n}) - (\mathbf{r}\times\mathbf{n})(\mathbf{H}\cdot\mathbf{B})\big)\,ds$$

Magneto-motive force (MMF): $$MMF = \int_L \mathbf{H}\cdot\mathbf{dl}$$

Flux Linkage: $$\Psi = \frac{1}{IS}\int_L \mathbf{A}\cdot\mathbf{J}ds$$

Example 2: Rotating Magnetic Field



We want: $B = B \sin(p\varphi)_{\max}$

In cylindrical coordinates:

$$B = rotA = \frac{1}{\rho}\frac{\partial A}{\partial \varphi}\vec{\boldsymbol{\rho}} - \frac{\partial A}{\partial \rho}\vec{\boldsymbol{\varphi}}$$

The rotating field is given by a proper Dirichlet boundary condition:

Magnitude: $|A| = B_{\max}$

Phase: $Arg(A) = \varphi \cdot p$

Example: Rotating Magnetic Field